METROECONOMICA
INTERNATIONAL REVIEW OF ECONOMICS

Metroeconomica 61:4 (2010) 676–682
doi: 10.1111/j.1467-999X.2010.04090.x

RESWITCHING AND DECREASING DEMAND FOR CAPITAL

Saverio M. Fratini*
Università degli Studi Roma Tre

(March 2009; revised November 2009)

ABSTRACT

We consider a Wicksellian or Neo-Austrian model of production with a continuum of techniques. For this model we provide an example in which a monotonically decreasing demand for capital schedule is combined with reswitching and a net product per worker that increases (over a certain interval) as the interest rate increases.

1. INTRODUCTION

According to Samuelson,[1] the most surprising and important consequence of the phenomenon of reswitching, i.e. the case in which a technique is in use for interest rates $\hat{r}$ and $\breve{r}$, but not for some r between the two, is the reversal of direction of the relation between the interest rate and the net product per worker. Reswitching is in fact a sufficient—albeit not generally necessary—condition for an increase in the net product per worker as the interest rate increases.

In cases with a finite number of alternative production techniques, at a switch point between two techniques, the one with the higher net product per worker must also have the higher demand for capital per worker in value terms (cf. Pasinetti, 1969, p. 527). In such cases, as a result, the unconventional relation between the interest rate and the net product per worker is always associated with an increasing demand for capital schedule.

* The author wishes to thank E. Bellino, G. Bloise, M. Corradini, J. Duffy and two anonymous referees for their useful comments and suggestions. The usual disclaimer applies for any remaining errors.

[1] Cf. Samuelson (1966, p. 577; 1976, pp. 14, 15). The same views are also expressed by other important authors. See in particular Bruno *et al.* (1966, p. 528) and Burmeister (1980, p. 118).

We instead consider here a simple Wicksellian model with a continuum of production techniques (section 2) and demonstrate in this case, by means of a numerical example (section 3), the possibility of a monotonically decreasing demand for capital schedule despite the occurrence of reswitching, and hence a non-monotonic relation between interest rate and net product per worker. The paper ends with a discussion of some characteristics and implications of this result (section 4).

2. THE MODEL

We consider a Wicksellian model of production[2] in which the capital employed to produce the only consumption good of the economy is 'genetically' considered in terms of dated quantities of labour. In particular, we assume that a unit of net product could be obtained by the employment of ℓ_0 units of direct labour with ℓ_1 and ℓ_2 units of labour dated respectively one and two periods before.

The labour terms ℓ_0, ℓ_1 and ℓ_2 clearly depend on the technique in use. It is specifically assumed that there are as many techniques as non-negative real numbers, and a real θ is therefore used to denote each technique, with $0 \leq \theta$. The labour terms are then assumed to be continuous functions of θ: $\ell_0(\theta)$, $\ell_1(\theta)$ and $\ell_2(\theta)$.

Given an interest rate level r and a wage rate w, the optimal technique is the one that minimizes the unit production cost of net output. The techniques in use will therefore be the solution to the following minimization problem:

$$\begin{cases} \min\limits_{\theta} w \cdot \ell_0(\theta) + w(1+r) \cdot \ell_1(\theta) + w(1+r)^2 \cdot \ell_2(\theta) \\ \text{s.t.:} \quad 0 \leq \theta \end{cases} \tag{1}$$

Since the solution of the above minimization problem depends exclusively on the rate of interest, we denote as $\theta(r)$ the technique in use at an interest rate r. At the same time, $\ell_t(r) = \ell_t(\theta(r))$, with $t = 0, 1, 2$, are the labour terms with the technique that minimize costs for an interest rate r.

Finally, assuming the consumption good as the numeraire and using y and v, respectively, to denote the net product per worker and the demand for capital per worker in terms of value, we have

[2] For an extensive analysis of the model outlined here, see in particular Wicksell (1977, pp. 158–66 and 196–206) and Burmeister (1980, pp. 144–54). See also Pasinetti (1977, pp. 89–92) and Kurz and Salvadori (1995, pp. 165–168).

$$y(r) = \frac{1}{\ell_0(r) + \ell_1(r) + \ell_2(r)} \tag{2}$$

$$w(r) = \frac{1}{\ell_0(r) + (1+r)\cdot \ell_1(r) + (1+r)^2 \cdot \ell_2(r)} \tag{3}$$

$$v(r) = \frac{y(r) - w(r)}{r} \tag{4}$$

3. THE NUMERICAL EXAMPLE

We shall consider a very simple example. Let us assume $\ell_0(\theta) = a/(b+\theta)$; $\ell_1(\theta) = \theta$ and $\ell_2(\theta) = c/(b+\theta)$, with a, b and c taken as given parameters.

In this case, from the solution of the cost minimization problem (1) we obtain[3]

$$\theta(r) = \max\left[0; \sqrt{\frac{a + c(1+r)^2}{1+r}} - b\right] \tag{5}$$

On positing $a = 3$ and $b = c = 1$, we therefore obtain the results presented numerically in Table 1 and graphically in figures 1 and 2, which show, respectively, the demand for capital schedule $v(r)$ and the net product schedule $y(r)$.

In particular, for the numerical example considered here, according to equation (5), the technique $\theta = 1$ is in use for two separate interest rates $r = 0$ and $r = 2$ (the data of our example were chosen precisely in order to obtain this result, see also footnote 5), while $\theta < 1$ for every $0 < r < 2$.

Moreover, in the numerical example discussed here, as can be easily verified,[4] the net product per worker y increases monotonically when θ moves from 0 to 1 and the U-shape stretch of the net product schedule represented in figure 2 is therefore unquestionably due to reswitching. We thus have a case in which the demand for capital schedule is monotonically decreasing despite the fact that reswitching occurs and the net product per worker tends to increase with the rate of interest over a certain interval.

[3] The possibility of reswitching can already be grasped analytically. If we assume that $1 \geq \sqrt{a+c} - b \geq 0$ and $a/c > 1$, equation (5) tells us that the technique $\theta = \sqrt{a+c} - b$ is in use for two distinct interest rates, namely $r = 0$ and $r = a/c - 1$. See Hatta (1976) for a necessary condition of reswitching in a Wicksellian model.

[4] In our numerical example we have $y = (1+\theta)/(4+\theta+\theta^2)$, and therefore $\mathrm{d}y/\mathrm{d}\theta > 0$ for every $\theta \in [0, 1)$.

Table 1. Choice of technique, net product and demand for capital per worker

Interest rate	*Tecnique in use*	*Wage rate*	*Net product*	*Demand for capital*
0.0	1.00000	0.33333	0.33333	0.22222
0.1	0.95634	0.31211	0.33323	0.21111
0.2	0.92354	0.29270	0.33300	0.20149
0.3	0.89939	0.27484	0.33274	0.19299
0.4	0.88225	0.25838	0.33252	0.18535
0.5	0.87083	0.24316	0.33235	0.17837
0.6	0.86414	0.22908	0.33224	0.17192
0.7	0.86137	0.21605	0.33219	0.16592
0.8	0.86190	0.20396	0.33220	0.16029
0.9	0.86519	0.19276	0.33225	0.15499
1.0	0.87083	0.18237	0.33235	0.14997
1.1	0.87845	0.17273	0.33246	0.14521
1.2	0.88776	0.16377	0.33259	0.14069
1.3	0.89851	0.15544	0.33273	0.13637
1.4	0.91050	0.14770	0.33287	0.13226
1.5	0.92354	0.14049	0.33300	0.12833
1.6	0.93748	0.13378	0.33311	0.12458
1.7	0.95221	0.12752	0.33320	0.12099
1.8	0.96759	0.12168	0.33327	0.11755
1.9	0.98355	0.11622	0.33332	0.11426
2.0	1.00000	0.11111	0.33333	0.11111
2.1	1.01686	0.10633	0.33332	0.10809
2.2	1.03408	0.10185	0.33327	0.10519
2.3	1.05161	0.09765	0.33319	0.10241
2.4	1.06938	0.09370	0.33308	0.09974

In a nutshell, the example shows that $\mathrm{d}v/\mathrm{d}r < 0$ is compatible with $\mathrm{d}y/\mathrm{d}r > 0$, a possibility that appears to have been previously overlooked.

4. CONCLUSIONS

The impact of a change in the interest rate on the demand for capital (in value terms) can be broken down into a 'real effect' and a 'price effect' (cf. in particular Bhaduri, 1966; Burmeister, 1976; Garegnani, 1984). The first is the

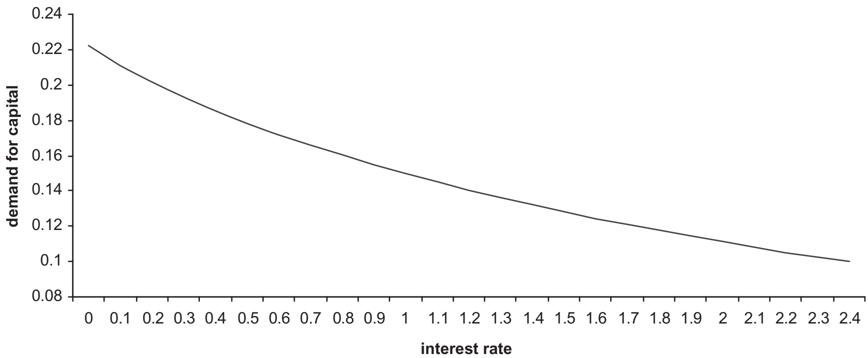

Figure 1. Demand for capital schedule.

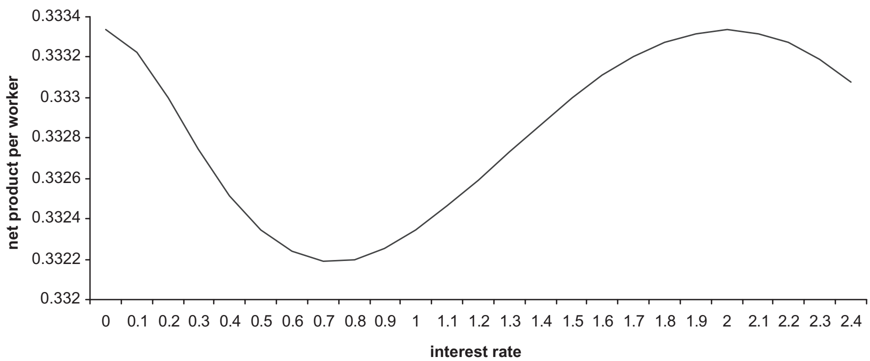

Figure 2. Net product schedule.

effect of a change in interest rate on the technique in use as well as the physical capital employed and the net product per worker obtained. The second is simply the effect on the value of the same physical capital caused by the change in the relative prices of capital goods associated with a variation in interest rate.

As defined at the outset, reswitching exclusively regards the real effect and can in this connection affect the shape of the demand for capital schedule. This shape also depends, however, on the price effect, which may be opposite in sign to the real effect and may prevail.

It is therefore possible to have a monotonically increasing demand for capital even in the case of a well-behaved choice of techniques and indeed a negative real effect. This possibility is well known and the simplest example is the case with a capital good whose production is more capital-intensive than that of a consumption good. Much less known is the possibility of a monotonically decreasing demand for capital associated with reswitching and indeed a positive real effect. This is the case examined here.

In our numerical example, there is a negative price effect prevailing over a positive real effect. As a result, the demand for capital schedule proves to be decreasing despite the reswitching, whereas the net product per worker increases with the interest rate after a necessary[5] initial decreasing stretch.

One further observation concerns the capital intensity of production and its measurement in terms of the value of capital employed per worker. The defects of this measurement are well known and some possible alternatives have been put forward.[6] Our example provides additional proof of the irrelevance of the capital–labour ratio. As we have seen, an increase in net product per worker may in fact be associated with a decrease in capital per worker and, as a result, the techniques that should be more 'productive', 'mechanized' or 'roundabout' (giving a larger product for the same employment of labour) instead appear to be those with lower capital intensity.

REFERENCES

Bhaduri, A. (1966): 'The concept of the marginal productivity of capital and the Wicksell effect', *Oxford Economic Papers*, 18, pp. 284–8.

Bruno, M., Burmeister, E., Sheshinski, E. (1966): 'Nature and implications of the reswitching of techniques', *Quarterly Journal of Economics*, 80, pp. 526–53.

Burmeister, E. (1976): 'Real Wicksell effects and regular economies', in Brown, M., Sato, K., Zaremka, P. (eds): *Essays in Modern Capital Theory*, North-Holland, Amsterdam.

Burmeister, E. (1980): *Capital Theory and Dynamics*, Cambridge University Press, Cambridge.

Garegnani, P. (1970): 'Heterogeneous capital, the production function and the theory of distribution', *Review of Economic Studies*, 37, pp. 407–36.

Garegnani, P. (1984): 'On some illusory instances of "marginal products" ', *Metroeconomica*, 36, pp. 143–60.

Hatta, T. (1976): 'The paradox in capital theory and complementarity of inputs', *Review of Economic Studies*, 43, pp. 127–42.

Kurz, H., Salvadori, N. (1995): *Theory of Production*, Cambridge University Press, Cambridge.

Mas-Colell, A. (1989): 'Capital theory paradoxes: anything goes', in Feiwel, G. (ed.): *Joan Robinson and Modern Economic Theory*, New York University Press, New York.

Pasinetti, L. L. (1969): 'Switches of technique and the "rate of return" in capital theory', *The Economic Journal*, 79, pp. 508–31.

Pasinetti, L. L. (1977): *Lectures on the Theory of Production*, Columbia University Press, New York.

Samuelson, P. A. (1966): 'A summing up', *Quarterly Journal of Economics*, 80, pp. 568–83.

[5] Because of the 'golden rule theorem' (cf. Mas-Colell, 1989, p. 509), when $r = 0$ the technique in use gives the maximum net product per worker—i.e. it is $\theta = 1$—and an increase in the rate of interest must therefore initially entail a decrease in the net product per worker.

[6] Cf. also Steedman, (2009, p. 151).

Samuelson, P. A. (1976): 'Interest rate determinations and oversimplifying parables: a summing up', in Brown, M., Sato, K., Zaremka, P. (eds): *Essays in Modern Capital Theory*, North-Holland, Amsterdam.

Steedman, I. (2009): 'Many capital–output ratios increasing with the interest rate: an industry-level analysis', *Metroeconomica*, 60, pp. 150–61.

Wicksell, K. (1977): *Lectures on Political Economy*, vol. I, A.M. Kelly, Fairfield.

Saverio M. Fratini
Department of Economics
Università degli Studi Roma Tre
via Silvio D'Amico, 77
00145 Rome
Italy
E-mail: fratini@uniroma3.it