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A Note on Reswitching and Intertemporal Prices

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ABSTRACT *Bliss (*Capital Theory and Distribution of Income*, Amsterdam/New York: North-Holland/Elsevier) claims that reswitching is nothing but an 'optical illusion' due to the exclusion of non-stationary price sequences from the analysis. This note develops this point. The standard case for choice of techniques and reswitching is reformulated in terms of Arrow-Debreu intertemporal prices and the conditions making these prices stationary are highlighted separately. It is then shown that the analysis of the choice of techniques in terms of 'switch points' requires stationary conditions.*

Keywords: prices; capital; reswitching

JEL Codes: B51; D21; D46

1. Introduction

Bliss (1975, pp. 235–244) showed in his seminal work Capital Theory and the Distribution of Income that the same consumption-and-accumulation path can be optimal for more than one intertemporal price system. He also claimed that 'reswitching'—the possibility of the same production technique being in use for interest rates lower than $\overset{\frown}{r}$ and higher than $\breve{r}$ but not for $r \in (\overset{\frown}{r}, \breve{r})$—arises when attention is confined to 'constant-rate-of-interest price systems' and disappears when price sequences not of this type are admitted. As he wrote:

> if a sequence of activities is an equilibrium for two price sequences, then it is an equilibrium for any convex combination of these price sequences. But if the two price sequences were constant-rate-of-interest price sequences for different values of the rate of interest a convex combination of them will not be a constant-rate-of-interest sequence and, as such, will be excluded from an analysis which confines its attention to constant-rate-of-interest sequences …

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> Surely part of the excitement caused by the demonstration that double switching was seemingly a fairly 'common' phenomenon, in the sense that the examples do not depend upon choosing very special values for the parameters, arose not from the fact that a technique could be chosen by firms at more than one value of the rate of interest, an obvious consequence of the sparseness of a finite linear production model, but because it seemed that the set of values at which a technique might be chosen was not connected. *But this is an optical illusion*. Firms do not choose plans in the light of the rate of interest alone. They choose plans in the light of complete intertemporal price systems (Bliss 1975, p. 239, original emphasis).

The purpose of this note is to clarify what Bliss maintains about reswitching without the complications of the intertemporal optimisation framework within which his arguments were set.[1] This will be done by means of a very simple model with two commodities and three methods of production, which can be combined to have two techniques.

Section Two considers the standard case in which the same stationary relative price for the two commodities is used on both the input and the output sides. This case, whose properties are well-known, is then re-written in terms of Arrow-Debreu intertemporal prices with some explicit conditions of stationarity imposed on them.

Section Three shows that even though there is in general a convex set of intertemporal prices for which the two techniques can be used simultaneously, no set of prices satisfying the stationarity conditions is convex.

Section Four defines the sets of intertemporal prices supporting the use of each technique or the coexistence of both and their sub-sets satisfying the stationarity conditions. The main conclusion drawn from this analysis is that the concept of a 'switch point' between two techniques makes no sense outside the hypothesis of stationary prices.

Finally, some technical conclusions are drawn in Section Five, and a brief account of the relevance of reswitching, and the results presented here, to the debate about capital theory is discussed in the Appendix.

2. Stationary Prices and Reswitching

Let us consider a model with two commodities, x and y. Each of the commodities is both a consumption and a (circulating) capital good. There are three methods of production, 1, 2 and 3:

[method 1]	a_{11} of x & a_{12} of y & l_1 of labour $\rightarrow$ 1 of x
[method 2]	a_{21} of x & a_{22} of y & l_2 of labour $\rightarrow$ 1 of y
[method 3]	a_{31} of x & a_{32} of y & l_3 of labour $\rightarrow$ 1 of y

[1]This point is also presented by Bliss (1990, p. 152) in his entry on 'equal rates of profit' in the *New Palgrave Dictionary of Economics*, but the argument there, rapidly sketched, is less clear than in the above quoted passage.

These three methods can be combined so as to obtain two techniques: technique α, using methods 1 and 2 and technique β, using 1 and 3.[2] Each technique is assumed to be viable and non-dominated.

On the assumption that the price of y in terms of x, denoted by π_y, is stationary, meaning that it does not change period by period, the characteristics of the choice of technique for the case addressed here are well-known, and we shall therefore confine discussion to some particular results. Specifically, as an implication of the 'non-substitution theorem', it is known that for every given interest rate r, taken within a certain interval, the methods used are determined, and the wage rate w and the price π_y, corresponding to r, are determined accordingly (cf. Burmeister and Dobell 1970, ch. 8, theorem 8, p. 242). Moreover, up to two switch points are possible in the two-commodity, three-method framework considered here (cf. Burmeister and Dobell 1970, p. 249–50). This means that the system:

$$(a_{11} + a_{12} \cdot \pi_y)(1 + r) + \ell_1 \cdot w = 1 \tag{1}$$

$$(a_{21} + a_{22} \cdot \pi_y)(1 + r) + \ell_2 \cdot w = \pi_y \tag{2}$$

$$(a_{31} + a_{32} \cdot \pi_y)(1 + r) + \ell_3 \cdot w = \pi_y \tag{3}$$

can have two economically meaningful solutions. Assuming that this is the case, if $\widehat{\pi} = [\widehat{r}, \widehat{w}, \widehat{\pi}_y]$ and $\breve{\pi} = [\breve{r}, \breve{w}, \breve{\pi}_y]$ are the price vectors solving the system, then one of the two techniques, say technique α, is in use (optimal) for interest rates in the intervals $[0, \widehat{r}]$ and $[\breve{r}, R]$_where R is the maximum rate of interest compatible with a non-negative wage rate—but not in the interval $(\widehat{r}, \breve{r})$. In other words, the set of interest rates—for which technique α is used—is not convex.

All this can be graphically represented, as in Figure 1, where there are the two wage-interest curves associated with techniques α and β, and, as is known, the technique used is the one that makes it possible to pay the maximum wage rate for a given rate of interest.[3] This is the usual way in which the reswitching argument is presented. However, for the purposes of the analysis developed in the following sections, some slight formal modifications are introduced.

First, instead of regarding the interest rate as the independent variable and the wage rate as the dependent one, it is possible to do the opposite and in so doing to say that, if $\widehat{\pi} = [\widehat{r}, \widehat{w}, \widehat{\pi}_y]$ and $\breve{\pi} = [\breve{r}, \breve{w}, \breve{\pi}_y]$ are the two solutions of the System 1–3, then technique α is in use for wage rate levels in the intervals $[0, \widehat{w}]$ and $[\breve{w}, W]$ but not $(\widehat{w}, \breve{w})$, and therefore the set of wage rate levels—for which technique α is in use—is not convex.

[2]This model corresponds to the one considered by Garegnani (1966, pp. 566–567) in the appendix of his article for the 1966 symposium in the *Quarterly Journal of Economics*.
[3]The wage-interest curve associated with technique α is the graph of the function $w\alpha = w\alpha(r)$, where $w\alpha$ solves the equations (1) and (2) for a given r. In the same way, a function $w\beta = w\beta(r)$ can be defined by the equations (1) and (3).

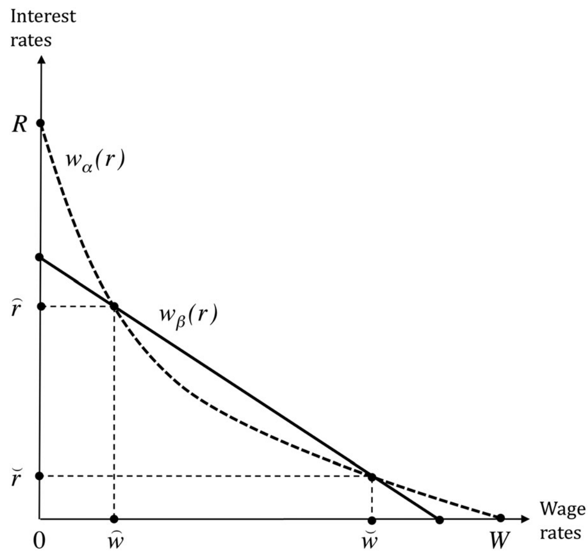

Figure 1. Wage-interest curves

Second, on the basis of our hypothesis of stationary relative prices, π_y is the price of y in terms of x for both the input and the output sides of Equations 1–3, despite the fact that inputs are commodities delivered in period t and outputs in period $t+1$. The hypothesis of stationary relative prices is implicit in Equations 1–3 but can be made explicit.

In particular, it is possible to follow the Arrow-Debreu methodology and distinguish commodities by their date of delivery and denote by $p_{i,j}$ the price of commodity i delivered in period j, suitably actualised, with $i = x$, y and $j = t$, $t+1$. Once this notation is introduced and the commodity x delivered in $t+1$ is assumed as the numéraire, i.e. $p_{x,t+1} = 1$ is posited, the following equality holds in the case of stationary relative prices considered here:

$$\frac{p_{y,t}}{p_{x,t}} = p_{y,t+1} = \pi_y \tag{4}$$

which can also be written as follows:[4]

$$p_{x,t} = \frac{p_{y,t}}{p_{y,t+1}} = 1 + r \tag{5}$$

[4]By definition, the own-rates of interest of the two commodities are: $r_x = p_{x,t}/p_{x,t+1} - 1$ and $r_y = p_{y,t}/p_{y,t+1} - 1$. It is therefore immediately evident that $r_x = r_y = r$ if and only if $p_{y,t}/px_{,t} = p_{y,t+1}/p_{x,t+1} = \pi_y$. Therefore, in a stationary price vector, 'all present-value price vectors [are] proportional to each other and all own-rates of interest equal' (Bliss, 1975, p. 69). See also Bliss (1990, p. 151).

It is, in fact, easy to prove that Equalities 4 and 5 together with the system

$$a_{11} \cdot p_{x,t} + a_{12} \cdot p_{y,t} + \ell_1 \cdot w = 1 \quad (6)$$

$$a_{21} \cdot p_{x,t} + a_{22} \cdot p_{y,t} + \ell_2 \cdot w = p_{y,t+1} \quad (7)$$

$$a_{31} \cdot p_{x,t} + a_{32} \cdot p_{y,t} + \ell_3 \cdot w = p_{y,t+1} \quad (8)$$

make it possible to obtain the usual System 1–3.

Finally, since the Systems 1–3 and 4–8 are actually equivalent, if the former has two solutions as is assumed here, then so does the latter.[5]

3. Stationary Prices and Non-convexity

Given the two solutions of System 1–3, namely $\widehat{\pi} = [\widehat{r}, \widehat{w}, \widehat{\pi}_y]$ and $\breve{\pi} = [\breve{r}, \breve{w}, \breve{\pi}_y]$, there are two intertemporal price vectors associated with them, namely $\widehat{p} = [\widehat{p}_{x,t}, \widehat{p}_{y,t}, \widehat{w}, 1, \widehat{p}_{y,t+1}]$ and $\breve{p} = [\breve{p}_{x,t}, \breve{p}_{y,t}, \breve{w}, 1, \breve{p}_{y,t+1}]$, that solve System 6–8 and satisfy Equations 4 and 5.[6]

Now, since System 6–8 is linear, it is clear that every convex linear combination of $\widehat{p}$ and $\breve{p}$ is again one of its possible solutions. In other words, every $\hat{p} = \theta\widehat{p} + (1-\theta)\breve{p}$, with $0 < \theta < 1$, solves the system. With $\widehat{p} \neq \breve{p}$, however, $\hat{p}$ does not satisfy Equations 4 and 5. This can be easily proved.

Proposition 1. Let p' and p'' be two stationary intertemporal price vectors associated with $\pi_y{}' \neq \pi_y{}''$ and $r' \neq r''$. Any price vector $p''' = \theta\, p' + (1-\theta)\, p''$, with $0 < \theta < 1$, is not a stationary price vector.

Proof. Since p' and p'' are stationary intertemporal price vectors, according to Equation 4 we have: $p'_{y,t}/p'_{x,t} = p'_{y,t+1}$ and $p''_{y,t}/p''_{x,t} = p''_{y,t+1}$. This implies: $p'''_{y,t+1} = \theta\, p'_{y,t}/p'_{x,t} + (1-\theta)\, p''_{y,t}/p''_{x,t}$. Therefore, since $p'''_{y,t}/p'''_{x,t} = [\theta\, p'_{y,t} + (1-\theta)\, p''_{y,t}]/[\theta\, p'_{x,t} + (1-\theta)\, p''_{x,t}]$, the price vector p''' can be stationary if and only if:

$$\theta \frac{p'_{y,t}}{p'_{x,t}} + (1-\theta) \frac{p''_{y,t}}{p''_{x,t}} = \frac{\theta p'_{y,t} + (1-\theta) p''_{y,t}}{\theta p'_{x,t} + (1-\theta) p''_{x,t}} \quad (9)$$

[5]Given a solution $\widehat{\pi} = [\widehat{r}, \widehat{w}, \widehat{\pi}_y]$ of System 1–3, Equalities 4 and 5 make it possible to determine the corresponding prices $\widehat{p}_{x,t}, \widehat{p}_{y,t}$ and $\widehat{p}_{y,t+1}$ univocally. In particular, we have: $\widehat{p}_{x,t} = 1 + \widehat{r}$; $\widehat{p}_{y,t} = \widehat{\pi}_y(1 + \widehat{r})$ and $\widehat{p}_{y,t+1} = \widehat{\pi}_y$. Therefore, since $\widehat{\pi} = [\widehat{r}, \widehat{w}, \widehat{\pi}_y]$ solves the System 1–3, then $\widehat{p} = [\widehat{p}_{x,t}, \widehat{p}_{y,t}, \widehat{w}, 1, \widehat{p}_{y,t+1}]$ satisfies Equations 6–8.

[6]This clearly means that $\widehat{p}_{y,t}/\widehat{p}_{x,t} = \widehat{p}_{y,t+1} = \widehat{\pi}_y$ and $\widehat{p}_{x,t} = \widehat{p}_{y,t}/\widehat{p}_{y,t+1} = 1 + \widehat{r}$; and similarly $\breve{p}_{y,t}/\breve{p}_{x,t} = \breve{p}_{y,t+1} = \breve{\pi}_y$ and $\breve{p}_{x,t} = \breve{p}_{y,t}/\breve{p}_{y,t+1} = 1 + \breve{r}$. See also the previous footnote.

After some simple algebraic steps, this can also be written as

$$(\pi'_y - \pi''_y)(r'' - r') = 0 \qquad (10)$$

which is, however, impossible as $\pi_y' \neq \pi_y''$ and $r' \neq r''$. ■

This result has some interesting implications. First, focusing solely on the stationary prices, the levels of the wage rate—at which the two techniques are simultaneously in use—are isolated points: the switch points. If instead non-stationary intertemporal price vectors are also admitted, it is not possible to talk about switch points because the two techniques can be simultaneously in use for every wage rate level between 0 and a certain maximum, provided that the other prices are adjusted according to Equations 6–8.[7]

Second, as shown by Proposition 1, in the case of two intertemporal price vectors that are stationary, any price vector obtained as a convex linear combination of them is not stationary. Consequently, any set of stationary intertemporal prices with more than one element cannot be a convex set.

Third, combining the previous two remarks, it is useful to distinguish two different kinds of non-convexity that can arise when the analysis is restricted to price vectors that are stationary, that is, that satisfy Conditions 4 and 5. There is the non-convexity of the wage rate levels at which a certain technique—technique α in the case here—is used due to the possibility of reswitching. There is, however, also the non-convexity of the set of stationary price vectors at which each technique—both α and β—is used due to the fact that no set of stationary price vectors can be convex, and this is independent of the reswitching phenomenon.

4. Sets of Price Vectors

Attention has so far focused primarily on price vectors that allow the two techniques to be used simultaneously. It is now possible to broaden our analysis to obtain a more general view.

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[7]The proof of the possibility of non-negative prices satisfying the System 6–8 with a null wage rate can be briefly outlined as follows. With $w = 0$, Equation 6 implies $p_{y,t} = 1/a_{12} - (a_{11}/a_{12})\, p_{x,t}$, which is the equation of a decreasing straight line crossing the vertical axis at point $1/a_{12}$. Equations 7 and 8 instead bring about $p_{y,t} = -(a_{21} - a_{31})/(a_{22} - a_{32})\, p_{x,t}$, which is the equation of a straight line starting from the origin of the axes. If, therefore, in the production of y, one of the two alternative methods employs more of commodity x per unit of output and the other more of commodity y, then the straight line derived from Equations 7 and 8 is increasing and consequently intersects the other line for a pair of strictly positive prices $(p_{x,t}, p_{y,t})$. And this also implies $p_{y,t+1} > 0$. If instead the method of production of y that employs more of commodity x per unit of output employs more of commodity y too, then there is a minimum strictly positive level of w compatible with the coexistence of techniques. There is a very simple reason for this, namely that the method employing both less of x and less of y per unit of output is unquestionably the most advantageous when $w = 0$. This appears, however, to be a particular case as it allows us to say which technique is more capital-intensive than the other through direct reference to physical capital without knowing the prices.

Since no activity can entail strictly positive profits in equilibrium, let us start from the set P, whose elements are intertemporal price vectors satisfying the following conditions:

$$a_{11} \cdot p_{x,t} + a_{12} \cdot p_{y,t} + \ell_1 \cdot w \geq 1 \tag{11}$$

$$a_{21} \cdot p_{x,t} + a_{22} \cdot p_{y,t} + \ell_2 \cdot w \geq p_{y,t+1} \tag{12}$$

$$a_{31} \cdot p_{x,t} + a_{32} \cdot p_{y,t} + \ell_3 \cdot w \geq p_{y,t+1} \tag{13}$$

If P_α is the set of price vectors for which the use of technique α is optimal, then $P_\alpha \subset P$ is made up of price vectors satisfying Conditions 11 and 12 with equality. In fact, since, for every $p \in P$, no activity can bring about strictly positive profits, then the optimal technique entails zero profits. In the same way, the set $P_\beta \subset P$ contains price vectors for which Conditions 11 and 13 are satisfied with equality.[8] Elements of the set $P_{\alpha\beta} = P_\alpha \cap P_\beta$ are clearly the price vectors for which the two techniques can be used simultaneously, that is, price vectors solving the System 6–8.

There are price vectors that are stationary in each of these sets. In particular, let $\tilde{P}_\alpha$, $\tilde{P}_\beta$ and $\tilde{P}_{\alpha\beta}$ be the sub-sets of P_α, P_β and $P_{\alpha\beta}$ whose elements satisfy the stationarity conditions 4 and 5. Since Conditions 11–13 are linear, the sets P_α, P_β and $P_{\alpha\beta}$ are convex. On the contrary, Proposition 1 implies that the sets $\tilde{P}_\alpha$ and $\tilde{P}_\beta$ are always non-convex. As regards the set $\tilde{P}_{\alpha\beta}$, its elements are the switch points and therefore, in the case addressed here, it can have (generically) one or two elements. In the latter case, it is non-convex.[9] With regard to the set of stationary prices $\tilde{P}_\alpha \cup \tilde{P}_\beta$, there is a one-to-one correspondence between its elements and wage rates in the interval [0, W].

Proposition 2. For any given $w' \in [0, W]$, there is one and only one price vector $p' = [p'_{x,t}, p'_{y,t}, w', 1, p'_{y,t+1}] \in P_\alpha \cup \mathrm{P}_\beta$ that satisfies the stationarity conditions 4 and 5.

Proof. As noted above in Section Two, for every given rate of interest $r \in [0, R]$, the methods of production used and the corresponding (zero-profit) price system $\pi = [r, w, \pi_y]$ are determined. Moreover, since there is a one-to-one correspondence between w and r under stationary conditions (the 'wage-interest frontier') then, given $w' \in [0, W]$, there is one and only one stationary price system $\pi' = [r', w', \pi'_y]$ associated with w'. Finally, given $\pi' = [r', w', \pi'_y]$, Conditions 4 and 5 make it possible to determine univocally the prices $p'_{\mathrm{x},t}$, $p'_{y,t}$ and $p'_{y,t+1}$ associated with r' and π'_y. ■

[8]It is clear that $P_\alpha \cup P_\beta \subset P$, and also that the set $P \setminus \{P\alpha \cup P_\beta\}$ is non-empty.

[9]The case in which the wage-interest curves of the two techniques coincide on a certain stretch is ignored here because it is associated with a non-generic choice of technical coefficients.

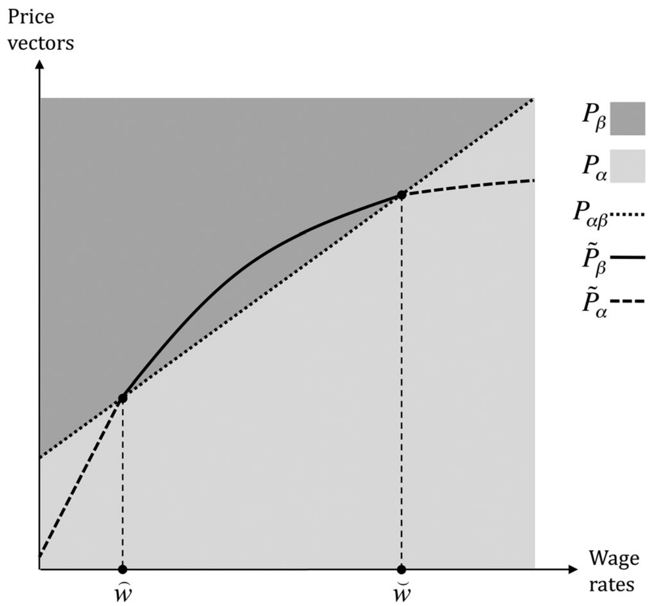

Figure 2. Sets of price vectors

To sum up, if non-stationary prices are ruled out, there is a unique price vector associated with any given level of the wage rate. In this context, talking about 'switch points' makes perfect sense, because the methods in use are determined once the wage rate level is known, and wage rate levels for which the two techniques can coexist are (generically) isolated points. If, instead, non-stationary intertemporal price vectors are admitted, then there is a point-set correspondence between wage rate levels and vectors in $P_\alpha \cup P_\beta$. In particular, given $w' \in [0, W]$,[10] it is possible for a set of intertemporal price vectors paying a wage rate w' to include: (a) some price vectors entailing the use of technique α alone; (b) other price vectors at which technique β only is in use; and (c) a price vector allowing both the techniques to be simultaneously used.[11] As a result, if the analysis is not restricted to stationary price sets, there are no 'switch points' because the two techniques can be used simultaneously for every wage rate level in a certain interval, provided that the other prices are properly adjusted.

All these considerations about price sets and wage rate levels can be exemplified—albeit not properly represented—by means of a graphic

[10]It can be observed that in the case of stationary prices, wage rate levels above W are regarded as economically inadmissible because they bring about negative interest rates r. When non-stationary price vectors are instead admitted, it seems possible to have vectors of non-negative prices associated with wage rates higher than W. This possibility appears, however, to play no role in the argument presented in this note.

[11]To see this it is sufficient to observe that, once w is taken as given, there are three unknown prices in the System 11–13 and therefore its degrees of freedom depend on the number of conditions satisfied with the inequality sign.

diagram with wage rate levels on one axis and price vectors on the other, as in Figure 2. [12]

In Figure 2, the set of stationary intertemporal price vector $\tilde{P}_\alpha \cup \tilde{P}_\beta$ is represented by a curved line, since, as seen above, there is a one-to-one correspondence between its elements and wage rate levels. Moreover, it is clear that the set $\tilde{P}_\alpha \cup \tilde{P}_\beta$ is not convex and therefore that no linear convex combination of two stationary intertemporal price vectors is a vector of stationary prices. The sets P_α (light grey), P_β (dark grey) and $P_{\alpha\beta}$ (dotted straight line) are instead convex. The two intersections between the sets $\tilde{P}_\alpha \cup \tilde{P}_\beta$ and $P_{\alpha\beta}$ are the switch points.

5. Conclusions

This note has the precise and limited purpose of clarifying the framework in which reswitching can occur and the reasons why it cannot occur in other frameworks. The importance of these results—especially for the purpose of developing critical arguments of general equilibrium theory—are briefly discussed in the Appendix.

On the basis of the analysis put forward in this paper, there is no doubt that the possibility of reswitching is restricted to cases in which the stationarity of relative prices is assumed or implied by features of the model. This conclusion follows primarily from the fact that, as we have seen, the concept of a switch point between two techniques loses its meaning outside the case of stationary prices. On the one hand, if non-stationary price vectors are admitted, then two techniques can be simultaneously used for every wage rate taken in a certain interval. On the other, it is under stationarity conditions alone that a wage rate level (taken in a certain interval) can be univocally associated with a price vector (Proposition 2) so that the technique used is simultaneously known. In the non-stationary case, on the contrary, many different price vectors, supporting the use of different techniques, can be associated with the same wage rate.

Moreover, no set of stationary intertemporal price vectors is convex (Proposition 1). The role of this fact in the reswitching phenomenon is not clear. It implies that the set of price vectors associated with the switch points between two techniques is made up of isolated points, but the central issue is whether this set has more than one element, and non-convexity does not appear to have much to do with this.

Finally, since reswitching requires the stationarity of relative prices period by period, it is obvious that it cannot take place in a framework where stationarity is inconceivable, such as atemporal equilibrium.

Acknowledgments

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[12] The idea for this kind of graphic representation is taken from Figure 17.D.6 in Mas-Colell, Winston and Green (1995, p. 598), where points on the vertical axis are assumed to be vectors of $L-1$ prices.

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Appendix: The Role of Reswitching in the Debates on the Theory of Capital

In order to clarify the relevance and importance of the arguments put forward in this note, let us briefly recall the role played by reswitching in the various debates related to the theory of capital over the last 50 years. It was indeed about half a century ago, during the debate on Samuelson's surrogate production function, that the possibility of reswitching arose, at least in the form discussed here.[13] In that case, it played an important part in proving that it is impossible—at least without running the risk of paradoxical results—to aggregate a set of heterogeneous capital goods into a 'jelly' and regard it as a factor of production to be employed together with labour and land. Despite its widespread use in mainstream macroeconomics, this 'aggregate capital', understood as a factor of production, is rather like the Loch Ness monster. Despite some alleged sightings, there is no real scientific proof of its existence.[14]

Moreover, the results of that debate, and in particular the possibility of 'reverse capital deepening', were used to criticise the neoclassical idea of the interest rate as a price establishing equilibrium between the demand of those wishing to borrow capital—intended as an amount of purchasing power—and the supply of those intending to lend it.[15] In particular, if the demand for capital is identified with the value of the capital goods that entrepreneurs wish to

[13]This refers here to the 1966 symposium on the *Quarterly Journal of Economics*.
[14]This very peculiar idea of aggregating a vector of quantities of different commodities into a scalar quantity of a factor of production appears to derive from confusion over two different concepts, namely 'capital' and 'capital goods', which are often erroneously seen as two sides of the same coin. Capital is a scalar quantity but not an input, being an amount of purchasing power used to anticipate the costs of production before revenues are obtained. For example, wages paid *ex ante* are 'advanced' from capital, whereas *post factum* wages are paid from revenue (Sraffa 1960, p. 10). Capital goods—whose costs, beside the others, are anticipated by capital—are inputs but cannot be aggregated without the risk of arriving at paradoxical results. In order to avoid any confusion, it would be better to use 'means of production' instead of 'capital goods' and drop the terms 'aggregate' and 'disaggregate' capital, as they do not correspond to rigorously scientific objects.
[15]As Marshall (1920, p. 534) wrote: 'interest, being the price paid for the use of capital in any market, tends towards an equilibrium level such that the aggregate demand for capital in that market, at that rate of interest, is equal to the aggregate stock forthcoming there at that rate.'

employ (with a given employment of labour), then reverse capital deepening may cause this demand to rise as the rate of interest increases, at least within a certain interval, and this may lead in turn to multiple and/or unstable equilibria.[16]

This argument was initially used against traditional versions of the neoclassical/marginalist theory such as Wicksell's but with only limited success, not least because Wicksell's theory also presents major logical flaws regarding the supply of capital, which is included, in value terms, among the data[17] as if it were the available quantity of a factor of production.[18] Consequently, the logical consistency of Wichsell's equilibrium was questioned and the issue of stability appeared less important. The problems regarding the endowment of capital in general equilibrium models—which also involve Walras' theory, albeit for opposite reasons—belong to another debate, however, and are not related to the present concern of reswitching.

Those involved in the debate from the neoclassical side changed in the mid-1970s.[19] As noted by Cohen and Harcourt (2003), Samuelson and Solow were replaced by Bliss and Hahn.[20] At the same time, the field of battle also changed with the scenario becoming that of the Arrow-Debreu equilibrium or, in more general terms, the neo-Walrasian approach. The scholars that Hahn called 'neo-Ricardians' were accused of aiming their critical arguments at 'vulgar theories of textbooks' instead of the 'current mainstream theoretical literature'. In particular, according to Hahn (1975, p. 363), their criticisms had 'no bearing on the mainstream of neoclassical theory simply because it does not use aggregation'.

The critics responded in two ways, the first of which concerned the meaning and relevance of the neo-Walrasian notions of equilibrium and in particular of the Arrow-Debreu intertemporal equilibrium. It was argued that—unlike the 'normal' equilibrium of Marshall, Wicksell and the other founders of the neoclassical/marginalist approach—the new notions of equilibrium, determining a different price system for every date of delivery of commodities, cannot be regarded as an explanation of the forces acting on the central position around which actual relative prices fluctuate[21] and are therefore devoid of practical relevance.[22]

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[16]An instability argument grounded on the increasing shape of the demand for capital schedule due to reverse capital deepening was already present in Garegnani (1970). Other relevant examples are Garegnani (1990) and Kurz and Salvadori (1995).

[17]For a discussion of the reasons why Wicksell was essentially forced to take the value of the existing stock of capital as given, see Garegnani (1990), Kurz (2000) and Fratini (2013b).

[18]See also Potestio (1999) and Kurz and Salvadori (2001).

[19]While the most relevant contributions of this second phase are Bliss (1975) and Hahn (1975, 1982), its beginning can perhaps be dated earlier and associated with the publication of Bliss's (1970) comment on Garegnani's article (1970) in the *Review of Economic Studies*.

[20]Stiglitz (1974) and Dixit (1977) can also be mentioned. They expressed views on the debate although they did not actually take part in it.

[21]The point is complex and cannot be given the attention it deserves here. Readers are therefore referred in particular to Garegnani (1976) and Petri (1978).

[22]This seems to be recognised even by neo-Walrasian authors when they write that the Arrow-Debreu equilibrium model is just a benchmark against which the real economy can be measured and not a state toward which it tends. See Hahn (1985) and Geanakoplos (1989).

The second has more to do with the arguments of the present note. It was maintained that the phenomena of reswitching and reverse capital deepening can affect the multiplicity and stability of equilibria in the neo-Walrasian models through the equilibrium between investments and savings, which is more or less implicit in those models. Although some attempts were made to use these arguments within the Arrow-Debreu framework—for example, Garegnani (2003) and Schefold (2005)—this note proves that reswitching cannot be found there.[23] Moreover, it is very doubtful that meaningful conceptions of saving and investment can be obtained in a model in which all the commodities, for every possible date of delivery, can (and must) be traded simultaneously in a single instant.[24] In any case, with reference to stationary equilibria of overlapping-generation models, it has been shown (see Fratini 2007, 2013a) that multiplicity and instability may arise because of reswitching, whereas the simple reverse capital deepening due to 'price Wicksell effects' does not seem to be equally important.

Finally, there are many useful contributions that have not been mentioned in this very schematic reconstruction of the debates connected with reswitching simply because they do not belong to a specific side in the debate. This is the case both of the articles and books devoted to reconstructing and summarising the subject of the debates, such as Harcourt (1972), and of works seeking to clarify technical aspects of reswitching, in which respect attention should be drawn to the contributions by Burmeister (1976, 1980).

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[23]See Mandler (2002, 2005) for a neoclassical reply to the arguments of Garegnani and Schefold.

[24]On the one hand, if outputs can be sold in the very moment when inputs have to be paid for, no investment of capital appears necessary because costs can be met directly out of revenues. On the other hand, as saving has the function of moving purchasing power from the present to a subsequent market day, it appears to have no role in a model where markets, for present and future delivery, are open for a single instant, so that the entire purchasing capacity of each household exists and is spent in that moment.