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1. Introduction

Regardless of whether it is prior or subsequent to that of 'marginal utility',[1] the idea of 'marginal productivity' unquestionably derived from a generalisation of the Ricardian theory of intensive rent, which is grounded on the possibility of applying successive doses of labour[2] on a fixed area of land and thus giving rise to successive increments in the amount of produce obtained. While Ricardo made use of this possibility in order to determine rents for a given wage rate, however, marginalist economists tried to use it for the determination of all the distributive variables.

As is well known, the major difficulty in this generalisation is capital. Unlike labour and land, most capital goods are highly specialised inputs invented and produced in order to perform a specific task in a specific way.[3] As a result, the change to a technique offering a higher or lower product per worker generally entails a change in the kind of capital goods

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1 According to Kauder (1965), the idea of marginal utility dates back even to Aristotle.
2 Although Ricardo referred to the application of successive doses of capital on a given area of land, the capital is assumed in his analysis to consist (essentially) of wages paid at the beginning of the process and the wage rate is taken as a given. Each dose of capital therefore corresponds to a dose of labour.
3 While multi-purpose capital goods do exist, e.g. Swiss Army penknives and computers, they can be regarded as exceptions.

employed. In other words, if there is no change in the latter, there can be no change in the former.

The explanation of distribution based on the marginal productivity of factors therefore requires that capital should be conceived as capable of changing its physical form while remaining fixed in terms of quantity.[4] Jevons, Böhm-Bawerk, J.B. Clark and many other economists accordingly attempted to build a marginalist theory of distribution by adopting a conception of capital based on the average period of production. This seemed to allow the possibility of an adjustment in the physical composition of the capital in use on the one hand and a measurement of the amount of capital independently of prices and income distribution on the other.

In order to elucidate the role played by the average period of production in the earlier versions of marginalist theory, let us imagine a world in which the only consumption good is obtained by the employment of labour during the T periods of time preceding the moment of output.[5] Let u_t, where $t = 1, 2, \ldots T$, be the share of labour employed t periods before output is obtained, so that $\sum_{t=1}^{T} u_t = 1$. The average period of production can then be defined by the following formula:

$$\theta = \sum_{t=1}^{T} t \cdot u_t. \tag{1}$$

The amount of output obtained per worker is then assumed to be a function of this average period of production $f(\theta)$, with $f'(\theta) > 0$ and $f''(\theta) < 0$.

Moreover, if simple interest is assumed at a rate r and w is used to denote the wage rate in terms of the consumption good, the cost of production per worker is:

$$c = \sum_{t=1}^{T} w \cdot u_t \cdot (1 + tr) = w \cdot (1 + \theta \cdot r) \tag{2}$$

4 According to a standard definition, for example, the marginal product of labour is the increase in output obtained from a given capital stock when an additional worker is employed. It is obvious here that the given capital stock cannot be regarded as a vector of physical quantities of capital goods – otherwise no change would be possible in the technique used and the output obtained – but necessarily as a given magnitude that can take different forms. On this point see also Trabucchi (2011).

5 Following both Wicksell and the analysis by Malinvaud discussed here, I am assuming that T is a finite number (integer). As will be made clear in section 2 (see footnote 9), this means that production is not circular.

For a given wage rate and interest rate, the optimal – i.e. profit-maximising – average period of production can therefore be found by solving the first-order condition:

$$f'(\theta) - w \cdot r = 0 \tag{3}$$

Moreover, extra profits must vanish under the hypothesis of free competition, and therefore:

$$f(\theta) - w \cdot (1 + \theta \cdot r) = 0 \tag{4}$$

Equations (3) and (4) make it possible to associate each possible interest rate r with a wage rate w and an average period of production θ. In particular, from Equations (3) and (4) we obtain:

$$\frac{f'(\theta)}{f(\theta) - f'(\theta) \cdot \theta} = r \tag{5}$$

And therefore, because of $f''(\theta) < 0$, a decrease in the interest rate entails a longer average period θ.[6] As Samuelson put it, this is 'the simple tale told by Jevons, Böhm-Bawerk, Wicksell and other neoclassical writers', according to which, 'as the interest rate falls in consequence of abstention from present consumption in favour of future, technology must become in some sense more ''roundabout'', more ''mechanized'', and more ''productive''' (cf. Samuelson 1966: 568).

As Samuelson claimed and as demonstrated, however, this simple tale is not 'universally valid'. To be precise, in the form presented here, it is clearly based on very strong assumptions, such as the application of the simple interest formula and the presence of a single primary factor (labour).[7]

It is precisely because of the strong assumptions required that Wicksell abandoned this conception of capital in his *Lectures* (1967 [1901]) after initially adopting it in *Value, Capital and Rent* (1970 [1893]). The rest of the story is well known and there is no need to tell it again here. We can thus proceed directly to more recent times, when Malinvaud attempted in a paper of 2003 celebrating Wicksell's contribution to the theory of capital to take up the idea of the average period of production introduced by Hicks in *Value and Capital* (1946) and *Capital and Time* (1973).

6 Let us posit $g(\theta) = \frac{f'(\theta)}{f(\theta) - f'(\theta) \cdot \theta}$. It follows that $g'(\theta) = \frac{f(\theta) \cdot f''(\theta)}{[f(\theta) - f'(\theta) \cdot \theta]^2}$ and $f''(\theta) < 0$ implies $g'(\theta) < 0$. From Equation (5) – i.e. $g(\theta) = r$ – the average period θ and the interest rate r must therefore vary in opposite directions.

7 For a discussion of the traditional average period of production, see also Garegnani (1960: 123–36) and Petri (2004: 99–117).

As reconstructed in the following section, Hicks defined the average period of production associated with a technique as depending on the rate of interest and therefore maintained that a meaningful ranking of techniques on the base of their average period can be obtained only for a given interest rate. Taking this idea and the underlying conception of the average period as his starting point, Malinvaud claimed that once techniques have been ranked according to the average period for a given initial rate of interest, a fall in the same entails the use of a technique with a longer average period, just as in the traditional neoclassical tale.

After a brief reconstruction of Malinvaud's argument (sections 2 and 3), we will show that the result is much less encouraging for the neoclassical theory than it might seem. The major problem is not the fact that a change in the interest rate affects the average period of production associated with a technique, despite Hicks' and Malinvaud's focus on this point, but rather that it affects the ranking of techniques. By using an example with two techniques (section 4), we will show that a rise in the rate of interest entails the use of a technique with a shorter average period even in the case of reswitching, simply because the ranking of techniques is inverted at the two switch points. This example also makes it possible to show that the technique with the longest average period of production may be the one with the lowest net product per worker, thus giving rise to serious doubts as to the real significance of the Hicks–Malinvaud average period of production.

2. The Hicks–Malinvaud average period of production

In our presentation of the average period of production as found in Hicks (1946, 1973) and Malinvaud (2003), we shall consider a simple model of production with n commodities, labelled 1, 2, 3, …, n, and assume that commodity 1 is the only final good.

In accordance with standard notation, a technique is denoted by $A \oplus \ell \rightarrow I$, where A is an $(n \times n)$ matrix of unit input coefficients and ℓ is an n-vector of labour coefficients – so that $a_{ij} \geq 0$ and $\ell_i > 0$ are, respectively, the quantity of the j-th commodity and the amount of labour employed in the production of one unit of the i-th commodity – and I is the $(n \times n)$ identity matrix.

For every given technique, the quantity y of commodity 1 obtained as net product per worker can be determined by solving the following system:

$$q \cdot [I - A] = y \cdot e_1 \tag{6}$$

$$q \cdot \ell = 1 \tag{7}$$

where q is the vector of gross products and e_1 is the (row) vector [1, 0, ..., 0]. Therefore, we have:

$$y = \frac{1}{e_1 \cdot [I - A]^{-1} \cdot \ell} \tag{8}$$

By the use of a well-known[8] linear algebra theorem, under standard assumptions about technology,[9] we can write:

$$[I - A]^{-1} = \sum_{t=1}^{T} [A]^{t-1} \tag{9}$$

And consequently, we have:

$$e_1 \cdot [I - A]^{-1} \cdot \ell = \sum_{t=1}^{T} e_1 \cdot A^{t-1} \cdot \ell = \sum_{t=1}^{T} x_t \tag{10}$$

where $\sum x_t$ is the total amount of labour embodied in one unit of commodity 1 and $x_t = e_1 \cdot A^{t-1} \cdot \ell$ is the quantity of labour (per unit of final output) employed t periods before the output emerges.

From Equations (8) and (10), it follows that:

$$y = \frac{1}{\sum_{t=1}^{T} x_t} \tag{11}$$

As y is the net product per worker of commodity 1, if we use u_t – as above – to denote the share of labour employed t periods before the final output is obtained, then $u_t = y \cdot x_t$ and, in accordance with Equation (11), $\sum u_t = 1$.

Given a wage rate w, paid at the beginning of each period t, and an interest rate ρ, the extra-profits per worker are:

8 Cf. Dorfman et al. (1958: 500, 501), Pasinetti (1977: 265, 266), Kurz and Salvadori (1995: 513, theorem A.3.3).

9 It is assumed in particular that A is ‘productive’ – i.e. the technology is such that levels of activity exist making it possible to obtain a strictly positive net output of each product – or equivalently that matrix A has no eigenvalue λ such that $|\lambda| \geq 1$.

As already mentioned, it is also assumed that production is not circular, which means that no capital good enters directly or indirectly into its own production. This implies that there exists an integer number $T < n$, such that $[A]^T$ is the $n \times n$ null matrix.

$$
\begin{aligned}
& y - w[u_1(1+\rho) + u_2(1+\rho)^2 + u_3(1+\rho)^3 + \cdots + u_T(1+\rho)^T] \\
& \quad = y - w\sum_{t=1}^{T} u_t(1+\rho)^t
\end{aligned} \tag{12}
$$

Therefore, if several techniques are available, the one in use will be the one maximising the difference in Equation (12).[10]

Following Malinvaud, let us denote by v_t, with $t = 1, 2, \ldots, T$, the labour shares associated with the profit-maximising technique at the given ruling wage rate and profit rate. The total cost per worker with the optimal technique is therefore, $w \cdot \sum v_t \cdot (1+\rho)^t$, while $w \cdot v_t \cdot (1+\rho)^t$ is the part of the same cost that can be ascribed to the employment of labour t periods before the output. Finally, the proportion of this part of the cost to the total – i.e. $v_t \cdot (1+\rho)^t / \sum v_t \cdot (1+\rho)^t$ – is the weight used in the Hicks–Malinvaud formula for the average period of production.

Therefore, according to the Hicks–Malinvaud conception, the average period of production is (cf. Malinvaud 2003: 516):[11]

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10 If the pair (w, ρ) is not on the wage–interest frontier – which can be defined for our purposes as the set of pairs (w, ρ) such that the maximum amount of extra-profit is zero – then the technique maximising the extra-profits per worker could be different from the one maximising the extra-profits per unit of capital. Malinvaud does not address this issue explicitly in his paper and determines the optimal technique by solving a problem of extra-profit maximisation for L workers, where L is the exogenously given supply of labour of the economy (cf. Malinvaud 2003: 515). Since constant returns to scale are assumed, however, the technique solving Malinvaud's problem is the one maximising the extra-profit per worker.

This point will be immaterial, however, for the argument presented in section 4, as the two pairs (w, ρ) considered there will be switch points and hence both on the wage–interest frontier.

11 As regards Hicks, he refers in *Value and Capital* (1946: 186) to a 'stream of payments' denoted as $(x_0, x_1, x_2, \ldots, x_v)$. If β is the discount factor for one period, the present value of the stream of payments, which Hicks calls its 'capital value', is $x_0 + \beta x_1 + \beta^2 x_2 + \ \ldots\ + \beta^v x_v$. The elasticity of this 'capital value' with respect to β is

$$
\frac{\beta x_1 + 2\beta^2 x_2 + \cdots + v\beta^v x_v}{x_0 + \beta x_1 + \cdots + \beta^v x_v}
$$

and this is what Hicks calls 'the *Average Period* of the stream'.

Now, if we interpret (i) the 'payments' either as employments of labour at different dates or as the corresponding wage bills and (ii) β as the interest factor instead of the discount factor – and in doing this, we of course invert the

$$\theta = \sum_{t=1}^{T} t \cdot \frac{v_t \cdot (1+\rho)^t}{\sum_{t=1}^{T} v_t \cdot (1+\rho)^t} = \frac{\sum_{t=1}^{T} t \cdot v_t \cdot (1+\rho)^t}{\sum_{t=1}^{T} v_t \cdot (1+\rho)^t} \tag{13}$$

3. The inverse relationship between the Hicks–Malinvaud average period and the interest rate

Comparison of the traditional average period formula – Equation (1) – with the Hicks–Malinvaud version – Equation (13) – clearly reveals that the weights are shares of labour in the former but shares of costs in the latter. As a result, while the first is completely independent of prices and distribution variables, the average period associated with a technique depends on the rate of interest in the second case. This appears to be the main concern of Hicks and Malinvaud. In particular, what the two authors are worried about is the problem of distinguishing between two different effects of a change in the rate of interest on the average period of production.

The rate of interest has in fact two tasks to perform: (i) it enters into the determination of the average period associated with the techniques; and (ii) it makes it possible to establish which technique is optimal and thus in use. A change in the interest rate therefore affects the average period of production in two ways, by involving a change both in the average periods associated with the various techniques and in the technique in use.

Following Hicks, Malinvaud intends to focus attention on the latter effect alone and separates it from the former.[12] More precisely, with reference to

chronological order of the payments so that x_ν becomes the first payment and x_0 the last one – we obtain Equation (13).

12 The procedure Hicks suggested in order to focus the attention on the effect of a change in the technique in use on the average period seems to have some similarities with the separation of a real effect from a price effect when considering the changes in the amount of capital in use due to a variation of the interest rate. Some implications of a variation in the interest rate are in fact kept separate from the others in both cases, and attention is focused primarily on the change in the technique in use.

Moreover, as is well-known (see Garegnani (1984) for an in-depth analysis), consideration of the real effect alone may lead to some 'illusory instances' and in particular to an illusory equality between the interest rate and a ratio that, although apparently similar to the marginal product of capital, is not of the same nature. This equality cannot therefore be used in order to obtain a schedule of demand for capital, as Garegnani showed. Similarly, as we shall argue, the inverse relationship between the change in the rate of interest and the change in the average period of production, calculated in Hicks's way, cannot be

Equation (13), in his analysis the change in the interest rate affects the labour shares v_t but is not allowed to affect the interest factors $(1 + \rho)^t$ for every $t = 1, 2, \ldots, T$. In the words of Hicks as quoted by Malinvaud:

> [i]f the average period changes, without the rate of interest having changed, it must indicate a change in the stream [of inputs]; but if it changes, when the rate of interest changes, this need not indicate any change in the stream at all. Consequently, even when we are considering the effect of changes in the rate of interest on the production plan, we must not allow the rate of interest which we use in the *calculation* of the average period to be changed (Hicks 1946: 220).

Using R to denote the interest factor $(1 + \rho)$, let us rewrite the average period formula as follows:

$$\theta = \frac{\sum_{t=1}^{T} t \cdot v_t \cdot R^t}{\sum_{t=1}^{T} v_t \cdot R^t} \tag{13'}$$

Now, assuming a change in the interest rate, if we focus attention on the change in labour terms v_t while keeping constant the interest factor R that appears in Equation (13′), we obtain the change in the average period $\hat{d}\theta$ that Malinvaud considers 'relevant for comparative analysis' (2003: 517):[13]

$$\hat{d}\theta = \frac{\sum_{t=1}^{T} (t - \theta) \cdot R^t \cdot dv_t}{\sum_{t=1}^{T} v_t \cdot R^t} \tag{14}$$

After a rather long and boring series of mathematical operations (see Appendix 1), Malinvaud arrives at the conclusion that the change in the average period $\hat{d}\theta$ must always be opposite in sign to the change in the rate of interest. In particular, he writes (2003: 518)

> [a] decrease in the real interest rate ρ [...] is associated with a lengthening of the average period of production, given what we mean by such lengthening

and comments:

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interpreted as the result of a well-behaved mechanism of factor substitutability, as Malinvaud instead appears to believe.

13 Note that, as in Malinvaud (2003), $\hat{d}\theta$ is used to denote the variation in the average period calculated in the same way as Hicks (which does not correspond to the usual differential $d\theta$), i.e. considering the change in the interest rate only in terms of the changes it induces in the optimal labour terms v_t.

> [i]t is interesting to know that the average period of production, *a measure of the degree of roundaboutness*, contra-varies with the interest rate. [Emphasis added]

Following the path opened up by Hicks, Malinvaud thus seems to have arrived back at the simple tale of the old neoclassical writers but within a decidedly more general framework. His result is, however, not exactly the same as the traditional one and, as will be shown in the next section, the Hicks–Malinvaud average period is in fact far from being 'a measure of the degree of roundaboutness' of production.

4. The case with two techniques

As pointed out above, the Hicks–Malinvaud average period of production associated with a given technique is generally a function of the rate of interest, and the average period may therefore change with no change in the technique in use. This fact and its possible implications are viewed by Hicks and Malinvaud as the main problem connected with the use of their idea of the average period. We shall see in this section, however, that the problem is decidedly more serious and concerns the ranking of techniques on the basis of Hicks–Malinvaud average period, which may change, as will be shown, with the interest rate.

Let us consider an example in which there is a given set Φ of possible techniques. For each technique $\phi \in \Phi$, the maximum wage rate that can be paid – according to Equation (12) – for a certain interest factor R is:

$$w^{\phi}(R) = \frac{y^{\phi}}{\sum_{t=1}^{T} u_t^{\phi} \cdot R^t} \tag{15}$$

where y^{ϕ} is the net product per worker with technique ϕ, and u_t^{ϕ} is the share of labour (with $\sum u_t^{\phi} = 1$) required, with technique ϕ, t periods before the final output is obtained.

By differentiating the wage rate $w^{\phi}(R)$, we obtain:

$$\frac{dw^{\phi}(R)}{dR} = -\frac{y^{\phi}}{R} \cdot \frac{\sum_{t=1}^{T} t \cdot u_t^{\phi} \cdot R^t}{\left[\sum_{t=1}^{T} u_t^{\phi} \cdot R^t\right]^2} = -\frac{w^{\phi}(R)}{R} \cdot \frac{\sum_{t=1}^{T} t \cdot u_t^{\phi} \cdot R^t}{\sum_{t=1}^{T} u_t^{\phi} \cdot R^t} \tag{16}$$

and since, according to the Hicks–Malinvaud conception, the average period of production associated with technique ϕ is:

$$\theta^{\phi}(R) = \frac{\sum_{t=1}^{T} t \cdot u_t^{\phi} \cdot R^t}{\sum_{t=1}^{T} u_t^{\phi} \cdot R^t} \tag{17}$$

and Equation (16) implies:

$$\theta^{\phi}(R) = -\frac{dw^{\phi}}{dR} \cdot \frac{R}{w^{\phi}(R)} \tag{18}$$

Equation (18) is particularly important in our argument. It clearly states that the average period of production associated with technique ϕ is equal to the elasticity of the wage rate w^{ϕ} with respect to the interest factor R, with the sign changed. In other words, for a certain interest factor, the technique with the most elastic wage–interest curve is the one with the highest average period of production.

In order to explore the consequences of the above result, let us assume the presence of just two techniques,[14] α and β. For each technique, according to Equation (15), there is a wage-interest curve $w^{\phi}(R)$, with $\phi = \alpha, \beta$. Let us further assume that R' is an interest factor such that $w^{\alpha}(R') = w^{\beta}(R')$. In other words, R' is a switch point. Then, because of Equation (18), $\theta^{\alpha}(R') > \theta^{\beta}(R')$ if and only if $-\, dw^{\alpha}/dR > -\, dw^{\beta}/dR$ in R'. In other words, the technique with the steepest wage–interest curve has the highest average period of production at a switch point.

Then, however, if we assume the existence of an interest factor R'' different from R' – say $R'' > R'$ – such that $w^{\alpha}(R'')=w^{\beta}(R'')$, the ranking of techniques based on the period of production calculated at R'' must be opposite to the one calculated at R', i.e. $\theta^{\alpha}(R'') < \theta^{\beta}(R'')$. This result follows very simply from the observation that if the wage–interest curve $w^{\alpha}(R)$ is steeper than $w^{\beta}(R)$ at the switch point R', then it must be less steep than $w^{\beta}(R)$ at the subsequent switch point, as shown in Figure 1. Therefore, according to Equation (18), we must have $\theta^{\alpha} > \theta^{\beta}$ at R' and $\theta^{\alpha} < \theta^{\beta}$ at R''.

Moreover, when R moves in a neighbourhood of a switch point, for interest rates (or factors) lower than the switch level, the technique in use is the one with the steepest wage–interest curve.[15] As a result, the technique with the flattest wage–interest curve comes into use for interest rates higher than the switch level.[16] This is Malinvaud's result, according to which an

14 For a discussion of a Neo-Austrian model with a continuum of techniques, see Fratini (2010).

15 In fact, as has been demonstrated, in cases like the one considered here, for a given interest rate or factor, the optimal technique is the one that makes it possible to pay the highest wage rate (cf. for example, Garegnani 1970 and Burmeister 1980).

16 This fact is represented by Hicks, in *Capital and Growth* (1965), in the following way:

> [w]hether or not they have multiple intersections, the wage curves that correspond to preferred techniques are always related, at the point of change-over,

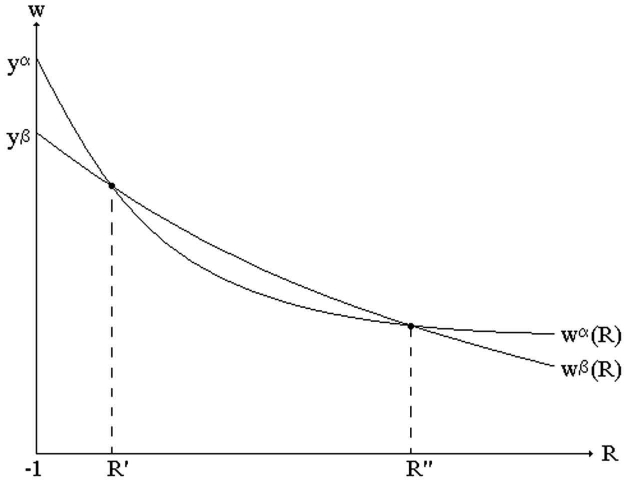

Figure 1. Wage-interest curves.

increase in the rate of interest is associated with the use of a technique with a shorter average period. And this is true at both switch points, since the technique with the highest average period at the interest factor R′ – i.e. technique α – is the one with the lowest average period at the interest factor R″. Therefore, despite the reswitching of techniques, thanks to the

in the same way. When there is a rise in the rate of real wages (or a fall in the rate of profit) there will always be a tendency of shift to a technique with wages curve which (in the way we have drawn our diagrams) is, at that, level of wages, a curve with a slope that is less. That is to say, the new wage curve must be one on which, at that level, profits are less affected by a given rise in wages. In that sense, and in that sense only, the new technique must be one with a lower labour-intensity. And since the whole thing can be put the other way, it is also a technique in which wages are more affected by a given rise in profits. In that sense, and only in that sense, we can safely say that the new technique is one of greater capital intensity (Hicks 1965: 166, 167).

The point is also considered in *Capital and Time* (1973), where Hicks writes: '[i]t is of course true that whenever a rise in the wage induces a change in technique, the change must always be such that, at the switch-point, the new efficiency curve has the greater slope' (Hicks 1973: 45), and then adds: '[i]t was this which I endeavoured to express, in the chapter of Value and Capital just cited [Ch. XVIII], in terms of a 'period of production' that was weighted by discounted values. There is nothing wrong in that treatment; but, except for its particular purpose, of criticising the 'old' Austrian theory, it is not very useful. It is better to go straight to the slopes (or elasticities) of the efficiency curves' (1973: 45, footnote).

Hicks–Malinvaud definition, a technique with a lower average period of production is adopted at both switch points as the rate of interest increases.

5. Conclusions

The founders of neoclassical theory developed the concept of the average period of production with the aim of using the principle of marginal productivity as a basis for the construction of the demand for capital function and then explaining the rate of interest in terms of the equilibrium of supply and demand.

It should be clear by now that the Hicks–Malinvaud average period of production cannot be used for this purpose. Its dependency on the interest rate has much more serious consequences than those mentioned by the two scholars. Variation in the interest rate affects not only the average period associated with each technique but also, and more importantly, the ranking of techniques as more to the less 'roundabout'.

As we have shown by means of the very simple example in section 4 with only two techniques and reswitching, the technique with the longest average period at the first switch point becomes the one with the shortest average period at the second. It is therefore impossible to say on the basis of the Hicks–Malinvaud formula which technique is the most 'roundabout' or 'capital intensive'. Consequently, their average period of production is not 'a measure of the degree of roundaboutness' of a technique, as is also shown by the fact that at the interest factor R', the technique with the longest average period – i.e. technique β – is paradoxically the one with the lowest net product per worker.

Malinvaud does not state explicitly why he finds it 'interesting [...] that the average period of production [...] contra-varies with the interest rate' (2003: 518). He appears to have no interest in building a demand for capital function[17] and the rate of interest is treated in his article as an independent variable (cf. Malinvaud 2003: 510 and 513). He writes,

17 It is worth noting that the word 'demand' never occurs in Malinvaud's article and 'supply' is only used (three times) with reference to the given endowment of labour L. The only passage in which he appears to refer to the supply of and demand for capital is the following:

> Capital theory has two sides; the consumption side, which comprises the ultimate choice between present and future goods, and the production side which, loosely speaking, results in a higher or lower marginal productivity of capital (Malinvaud 2003: 507).

He then adds, however, that in accordance with his view of Wicksell's analysis, his attention is focused on the production side alone.

however, that the Wicksellian model considered makes it possible to stress 'the true nature' of 'substitutions over time' (2003: 523). He therefore gives the impression of believing that a difference in sign between the change in the interest rate and the change in the average period (if properly calculated) is indicative of a well-behaved process of technique selection or of factor substitutability, which is the same thing.[18]

Our example proves that, on the contrary, Malinvaud's result in no way rules out phenomena like reswitching and can indeed coexist with them through inversion in the ranking of techniques.

In conclusion, the inverse relationship that Malinvaud finds between a variation in the interest rate (or factor) and the change in the average period associated with the optimal technique, if evaluated at the initial rate of interest, appears to have very little significance, if indeed any. In particular, our discussion of the Hicks–Malinvaud average period of production seems to confirm the following assertion made by Samuelson:

> [t]here often turns out to be no unambiguous way of characterising different process as more 'capital intensive,' more 'mechanized,' more 'roundabout,' except in the *ex post* tautological sense of being adopted at a lower interest rate and involving a higher real wage (1966: 582, 583).

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18 Malinvaud's article includes just one very brief reference to 'the Cambridge (England) movement of the 1950s and 1960s' (2003: 523), and this is in connection with the 'Wicksell effect', which Malinvaud addresses, quoting Swan (1956), as 'nothing but an inventory revaluation'. This appears to confirm that in Malinvaud's view, once the effects of the change in interest on values (inventory revaluations) are neglected, the choice of technique leads to well-behaved results. Surprisingly enough, the possibility of reswitching does not appear to have been contemplated.

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Appendix 1

In order to obtain the inverse relationship between the rate of interest and the average period of production discussed by Malinvaud, let us start, following the author, from the first-order conditions of the profit maximisation problem. The profits that firms intend to maximise are expressed by the difference in Equation (12), and the first-order conditions are therefore:

$$\frac{\partial y}{\partial v_t} = w \cdot R^t, \quad t = 1, 2, \cdots, T. \tag{A1}$$

Differentiation of Equation (A1) then gives us:

$$\sum_{s=1}^{T} \frac{\partial^2 y}{\partial v_t \partial v_s} \cdot dv_s = R^t \cdot dw + w \cdot t \cdot R^t \cdot \frac{dR}{R} \tag{A2}$$

Moreover, since extra profits must be zero in equilibrium, Equation (12) also implies:

$$w = \frac{y}{\sum_{t=1}^{T} v_t \cdot R^t} \tag{A3}$$

and then:

$$\frac{dw}{dR} = -\frac{w}{R\theta} \tag{A4}$$

Using Equation (A4) within Equation (A2), we get:

$$\frac{1}{w} \cdot \sum_{s=1}^{T} \frac{\partial^2 y}{\partial v_t \partial v_s} \cdot dv_s = (t - \theta) \cdot R^t \cdot \frac{dR}{R} \tag{A5}$$

which implies:

$$\frac{1}{w} \cdot \sum_{t=1}^{T} dv_t \cdot \left(\sum_{s=1}^{T} \frac{\partial^2 y}{\partial v_t \partial v_s} \cdot dv_s \right) = \frac{dR}{R} \sum_{t=1}^{T} dv_t \cdot (t - \theta) \cdot R^t \tag{A6}$$

Now, because of the non-increasing returns to scale assumption, we have:[19]

$$\sum_{t=1}^{T} dv_t \cdot \left(\sum_{s=1}^{T} \frac{\partial^2 y}{\partial v_t \partial v_s} \cdot dv_s \right) < 0 \tag{A7}$$

and therefore:

$$\frac{dR}{R} \sum_{t=1}^{T} dv_t \cdot (t - \theta) \cdot R^t < 0 \tag{A8}$$

19 If the production function $y = f(u_1, u_2, \ldots, u_T)$ exhibits non-increasing returns to scale, its Hessian matrix H, at the point $v = [v_1, v_2, \ldots, v_T]$, is negative semidefinite, that is $z \cdot Hz \leq 0$. Moreover, if H has rank T – 1, there is just one (non-null) vector z such that $Hz = 0$, and it is collinear to v. Then, since $dv = [dv_1, dv_2, \ldots, dv_T]$ cannot be collinear to v, $dv \cdot H\, dv$ is certainly negative.

which, in view of Equations (A3) and (14), implies:

$$\frac{y}{w} \cdot \hat{d}\theta \cdot \frac{dR}{R} < 0 \tag{A9}$$

so that, in conclusion, $\hat{d}\theta$ and dR must be opposite in sign.

Abstract

Malinvaud took up the concept of the average period of production introduced by Hicks in *Value and Capital* and then *Capital and Time*, in an article of 2003 celebrating Wicksell's contribution to the theory of capital, where he observed that once techniques are ranked according to the average period for a given initial rate of interest, a rise in the rate of interest entails the use of a technique with a shorter average period. After a brief reconstruction of Malinvaud's argument, it is shown that the result is far less encouraging for neoclassical theory than it might seem. The most important problem is not the fact that change in the interest rate affects the average period of production associated with a technique, despite the concern this aroused in Hicks and Malinvaud, but rather that it affects the ranking of techniques. An example with two techniques is used to show that a rise in the rate of interest entails the use of a technique with a shorter average period even in the case of reswitching simply because the ranking of techniques is inverted at the two switch points.

Keywords

Average period of production, capital, interest rate, Wicksell