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Sraffian indeterminacy: a critical discussion

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This paper is aimed at discussing Mandler's interpretation of Sraffa's price theory. In particular we will analyse Mandler's idea that an institutional determination of distribution, suggested by Sraffa, could be solidly advanced only in the case of equilibrium price indeterminacy in intertemporal sequential models. First it will be shown that this kind of indeterminacy arises from an arbitrary use of the tendency to a uniform rate of return on the supply prices of capital goods. Second, it will be remarked that, when Sraffa's contribution is placed, as it should, within the classical theory of value and distribution, no price or Sraffian indeterminacy will result. Finally, we will argue that Mandler's emphasis on the non-arbitrariness of the capital goods endowments, which is at the root of his indeterminacy result, naturally leads to refer to those normal positions of the economy whose only possible consistent determination is Sraffa's price theory.

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1. Introduction

In the last few decades indeterminacy has attracted increasing interest among general equilibrium theorists. While, on the one side, the generic determinacy of equilibria has been proved for the standard Arrow–Debreu framework (Debreu, 1970; Mas-Colell, 1975; Kehoe, 1980), on the other side, several robust cases of indeterminacy have been shown and analysed within different frameworks. In particular, most of the attention has been dedicated to equilibrium models over an infinite number of time periods, either with immortal households or with overlapping generations, where non-stationary path indeterminacy can arise even in the pure exchange case.

Although in these cases indeterminacy is closely linked to the endless time horizon hypothesis, and to the infinite number of commodities and markets it brings about (see Geanakoplos, 1987; Kehoe and Levine, 1990), the phenomenon has sometimes been called

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'Sraffian indeterminacy' (e.g. Geanakoplos, 1980; Borissov, 2002). The rationale for the use of this expression seems to stem only (or at least mostly) from Sraffa's idea of determining distribution from outside the production system: not in terms of equilibrium between the factors demand and supply, but rather in terms of social and institutional conditions.

There is, therefore, a 'Sraffian indeterminacy', which, in the sense specified above, has been admitted by neoclassical economists within the infinite horizon framework but has been denied for the cases having a complete and finite system of markets. For the latter, the generic determinacy of equilibria has been viewed as the proof that Sraffa obtains a degree of freedom in his system by some erroneous or *ad hoc* assumptions, in particular by the illegitimate omission of demand functions and of market-clearing conditions for goods and factors of production (cf. Arrow, 1968[1983]; Bliss, 1975; Hahn, 1982; Burmeister, 1984).

However, this result of determinacy was recently questioned by Mandler. Considering a sequential model where, after the initial period, capital endowments will not be arbitrarily given, he shows that indeterminacy can occur even in the case with a finite time horizon. Since it emerges by assuming a linear technology and would lead to income distribution being regulated by non-market institutions (Mandler, 2002, p. 215), he again calls it 'Sraffian indeterminacy'. Indeed, Mandler goes even further. By sharing the position in which Sraffa's analysis was based on *ad hoc* assumptions—such as, specifically, the ejection of demand functions from the system determining relative prices—he views his indeterminacy result as representing the sole possible way of defending Sraffa's idea of explaining distribution by 'extra-market forces'.

The aim of this paper is to examine the economic meaning of Mandler's sequential indeterminacy and to critically discuss the idea itself of the existence of a 'Sraffian indeterminacy'. In Section 2 we will make an in-depth reconstruction of Mandler's arguments, emphasising their similarities with other neoclassical interpretations of Sraffa's theory. In Section 3 we will go on to illustrate how Mandler's indeterminacy arises from an *ad hoc* use of the principle of the tendency to a uniform rate of return on the supply price of capital goods.[1] Finally, in Section 4, we will maintain that Mandler's representation of Sraffa's work in terms of a special case of general equilibrium is erroneous and misleading. We will also argue that actually no indeterminacy can occur in Sraffa when his contribution is situated, as it should be, within Smith's and Ricardo's surplus approach to value and distribution. In doing so we do not add anything particularly new on analytical grounds, but mainly collect and emphasise points rebutting critics of Sraffa that Mandler seems to have overlooked in his arguments. Indeed, as we will show, his emphasis both on indeterminacy and on the non-arbitrariness of capital endowments should naturally lead to Sraffa's aim of reconstructing economic theory on grounds different from those followed since the end of the nineteenth century with the spread of the neoclassical approach.

2. Mandler's Sraffian indeterminacy

Let us begin with an in-depth reconstruction of Mandler's arguments. This is needed since his views on indeterminacy are scattered and given prominence in many of his articles and summarise many widespread misunderstandings on classical theory and its revival by Sraffa (1960).

[1] This will be argued in the text using economic reasoning, while analytical details will be given in the Appendix.

In interpreting *Production of Commodities by Means of Commodities*, Mandler starts by emphasising the presence of a degree of freedom in Sraffa's price system when *all* the distributive variables are considered as unknowns to be determined. In particular, Mandler (1999A, p. 693) writes:

> Near the beginning of his *Production of Commodities by Means of Commodities* (1960), Sraffa specifies a system of equations that prices must obey if the capital invested in each sector of an economy is to earn the same rate of profit. The system contains one more unknown than equations, leading Sraffa to conclude that prices, and wage and interest rates in particular, are indeterminate. Sraffa's book has since been read as arguing that some outside, non-economic consideration must therefore set prices and distribution of income.

Therefore, in Mandler's view, price indeterminacy is a prerequisite for explaining income distribution on the basis of social-historical elements, that is, using his words, of 'non-economic considerations'. According to this view, the conflict between Sraffian (or classical) and neoclassical theories of distribution happens to rest on the presence of this degree of freedom. The generic determinacy of Arrow–Debreu equilibria thus appears as a neoclassical reply to Sraffa:

> Under weak conditions, the Arrow–Debreu model has at least one equilibrium and equilibria typically are determinate or locally unique. These results hold even for intertemporal models with linear production activities, the very setting of Sraffa's work. (Mandler, 2002, p. 204)

With this idea of the controversy between conflicting theories of value and distribution in mind, Mandler tries to test for cases in which the indeterminacy he calls 'Sraffian' may arise and others in which the market clearing conditions—that Sraffa would have neglected—can close the system without leaving any degree of freedom available.

For the sake of simplicity, in order to follow Mandler's reasoning, let us exclude joint production cases. Let us assume, moreover, that the wage rate is above the subsistence level, and that, if treated as a single variable, it cannot be included in the production coefficients of commodities in the same way as 'fuel for machines or forage for livestock'. By adopting the usual symbols, let us use B to denote the $(n \times n)$ diagonal matrix of outputs; p is the price vector of the n commodities; A is the $(n \times n)$ matrix having the inputs of each sector on the rows; L is the vector of labour inputs; r is the rate of profits and w the wage rate (paid post-factum). Given the production methods of the n commodities (i.e. given B, A and L), under free competition, we have the following system of price equations:

$$\mathbf{B} \cdot \mathbf{p} = (\mathbf{1}+\mathbf{r})\mathbf{A} \cdot \mathbf{p} + \mathbf{wL} \qquad (1)$$

which, once a numeraire is taken, gives us a unique price vector and a unique rate of profits, for any wage rate between zero and a certain maximum.

Referring to this system—which appears in chapter two of *Production of Commodities*—Mandler's first step is to renew, against Sraffa, the theses that some authors, like Jevons (1871) and Walras (1874[1954]), had already expressed with reference to the 'classical economists', and to Ricardo in particular. The first of these is that, once a commodity is taken as numeraire, if the wage rate is not at the subsistence level (so that it cannot be included within the means of production), we have a system with n equations and $n + 1$ unknowns ($n - 1$ relative prices, the wage rate and the profit rate) and, therefore, a situation of price indeterminacy (Mandler, 1999A, p. 696). The second thesis refers to the degree of indeterminacy, which would be even greater when it is maintained that produced quantities

cannot be considered as given, as in Sraffa, but are rather unknowns to be determined.[2] Mandler notes, in fact, that in Sraffa's price equations there is no mention 'of the demand for commodities, endowments, or the equilibrium of demand and supply', so that the solution of that system 'may not be consistent with market clearing' (Mandler, 1999A, p. 696).

A similar argument had already been used by Walras in order to maintain the necessity of demand functions in price determination. Indeed, if the produced quantities could not be considered as known before the prices are determined, then system (1) would have, even for a given wage rate, n equations and $2n$ unknowns (n produced quantities, $n - 1$ relative prices and the rate of profits). Therefore, there would be a 'missing equations' situation, whose nature seems similar to the one Walras (1874[1954], p. 425) was referring to:

> the English economists are completely baffled by the problem of price determination; for it is impossible for I to determine P at the same time that P determines I. In the language of mathematics one equation cannot be used to determine two unknowns

where P is, in Walras, the value of (aggregate) product excluding rents, I is the amount of profits, and the equation considered is $P = S + I$, where S is the amount of wages.[3]

Actually, assuming the constant returns to scale that characterises Walras' analysis, given the wage rate and the methods of production, the system can univocally determine relative prices independently of the quantities produced (cf. also Arrow and Starret, 1973[1983], p. 229). However, what Walras had in mind, and Mandler re-proposes as the 'neoclassical criticism' to Sraffa[4] is the following argument: relative prices and the rate of interest can be uniquely determined, under constant returns to scale, only for a given wage rate level, but these relative prices generate the demanded quantities of commodities and thus, indirectly, a certain demand for labour, and such a demand for labour may be inconsistent with its supply, bringing about a change in the wage level and, consequently, in the relative prices. Therefore, in this view, it is *necessary* to introduce the demand functions and the market-clearing conditions in order to determine the final level of relative prices.

Here is what Mandler considers as the first mistake made by Sraffa: that of taking the produced quantities as given magnitudes. Rather than being the signal that we are dealing with a different theory of value, which is not based on demand and supply, Sraffa's procedure is actually quite surprising to Mandler. Since Sraffa develops his theory in a framework of free competition and producer rationality (i.e., producers minimise their unit costs), he could not avoid accepting all the hypotheses of neoclassical theory (cf. Mandler, 1999A, p. 693). Therefore, the degree of freedom we find in his equations should not exist. In Mandler's words (1999B, p. 48):

[2] The relevance to this argument of the hypotheses concerning returns to scale will be discussed shortly.

[3] We find the same claim in Jevons. According to Jevons (1871, p. 269), Ricardo's attempt to obtain the wage–profit relationship from the equation 'Production = Profits + Wages' would be 'radically fallacious', being based on 'the attempt to determine two unknowns from one equation' (see also Wicksell, 1934, pp. 24–26). To some extent, the indeterminacy of Ricardo's theory of value was neglected by marginalist authors only when ascribing to him constant cost in manufacturing, a subsistence wage and demand in a 'logistical sense', i.e. as fixed by the necessaries required by the economic system (cf. Stigler, 1952).

[4] Although Mandler usually refers to the 'neoclassical critics' when he deals with the mistakes Sraffa is alleged to have made, he appears to share those criticisms. Thus, as for the market clearing conditions, Mandler (1999A, p. 694) writes: 'Sraffa does not discuss whether demand must equal supply; so this requirement may well be compatible with his economic philosophy'.

if we incorporate preferences and demand into Sraffa's *long-run framework*, the neoclassical mechanism for determinacy will *close* the model: if w, r or goods prices were to deviate from an equilibrium configuration, the long-run demand for labour would change, violating market-clearing.[5]

The above-mentioned 'long-run framework' brings us to the second fallacy Mandler ascribes to Sraffa: that of not distinguishing prices by date and, in so doing, of confining his analysis to a stationary state, that is to an equilibrium framework that is 'inhospitable to indeterminacy' (Mandler, 1999A, p. 694; cf. also 1999B, pp. 47–48).

According to Mandler, as Sraffa does not distinguish relative prices by date and does not have arbitrary initial endowments, he would thus be referring to the relatively unrealistic case of stationary state (or the similar one of a balanced growth path). But in this case, income distribution must be adjusted in order to have zero net savings per worker, so as to maintain the labour to capital ratio, and thus the income distribution itself, constant over time, thereby allowing relative prices to stay constant as well. Therefore, Mandler concludes, in a stationary state, once we have a net saving function—i.e. demand and supply functions (cf. Fratini, 2007, p. 49)—no indeterminacy can arise.

Conversely, dropping the stationary hypothesis and considering that 'changes through time in quantities will generally be inconsistent with stable relative prices' (cf. Mandler, 1999B, p. 32), allow Mandler to maintain that: (i) without demand and supply functions, the system would be underdetermined even for a give wage rate; and (ii) despite the introduction of demand and supply functions and market-clearing conditions, the indeterminacy case he calls 'Sraffian' can arise.

The first point is almost trivial. Assuming that every firm breaks even, and given the methods of production, denoting with $\mathbf{p}^0$ and $\mathbf{p}^1$ the n dimension (non-actualised) price vectors of commodities delivered, respectively, at date 0 and 1, we have the following system:

$$\mathbf{B} \cdot \mathbf{p}^1 = (\mathbf{1}+\mathbf{r})\mathbf{A} \cdot \mathbf{p}^0 + \mathbf{wL} \tag{2}$$

where, even if we take B, A, L and the wage rate w as givens, and the first commodity as numeraire both in period 0 and in period 1 (i.e. we posit $p_1{}^0 = p_1{}^1 = 1$), we will have n equations and $2n - 1$ unknowns (namely: the profit rate r, $n - 1$ relative prices of commodities delivered in period 0 and $n - 1$ relative prices of commodities delivered in period 1). Therefore, according to Mandler, in this case, we would be forced to address the neoclassical theory, in the form of Arrow–Debreu general equilibrium, or, by introducing price expectations, of temporary general equilibria. Indeed, with relative prices distinguished by date of delivery of commodities, the lack of market clearing conditions actually generates a kind of 'non-Sraffian' indeterminacy, as it were, in the sense that it will persist even with an exogenously given wage rate (or profit/interest rate).[6]

[5] We will argue later that the 'long-run position' should not be confused with the 'stationary state', and that in Sraffa there is no idea of given factor endowments, nor any necessity of introducing 'demand conditions'. Moreover, the treatment of outputs as givens—a characteristic of the classical economists tradition about price theory Sraffa refers to—does not prevent the clearing of *commodity* markets. In fact, according to Smith's analysis, the natural prices are associated with a situation in which the quantities of commodities 'brought to the market' equal their 'effectual demands'.

[6] In this case r should be intended as the own-interests rate of the numeraire commodity.

With reference to the second point, it is precisely the intertemporal setting of the economy that, in Mandler's view, allows the 'Sraffian indeterminacy' to arise in spite of the introduction of market-clearing conditions.

Indeed, in order to argue the possibility of 'Sraffian indeterminacy' within a neoclassical framework, Mandler considers a sequential equilibrium model over two time periods. In the first period markets are assumed to be incomplete, so that agents can trade the commodities delivered in that period, consumption and capital goods, but not the second-period consumption goods. At the beginning of the second period markets are opened again in order to allow agents to purchase consumption goods that are produced and delivered during that period.[7] Thus Mandler writes:

> In the first period, agents face the portion of the Arrow–Debreu price vector that corresponds to first period goods and unanimously anticipate that the remainder of Arrow–Debreu price vector, corresponding to second-period goods, will become market prices in the second period. We will assume that there are some intertemporal production activities which use first-period goods as inputs and yield second-period goods as outputs. Agents are able in the first period to purchase the outputs of these intertemporal activities for delivery in the second period.
>
> In the second period, agents start out with their natural second-period endowments plus deliveries from the intertemporal activities. [. . .]
>
> From the vantage point of the second period, the first period can be thought of as the pre-history of an Arrow–Debreu model, with the exception that now the endowments of agents in the second period are endogenously determined. (Mandler 1995, pp. 407–8)

More clearly, Mandler is assuming that, over the two periods, three kinds of production process take place: (a) the production of consumption goods delivered in the first period, which is carried out entirely within it; (b) the production of capital goods delivered at the beginning of the second period, which starts in the first period and ends at the beginning of the second one; (c) the production of consumption goods delivered in the second period, which takes place entirely within it. With the expression 'intertemporal production activities' Mandler is therefore referring to production processes of type (b).

The first-period equilibrium will determine the prices of initially available inputs and the prices of outputs coming from processes (a) and (b), which are consumption goods delivered in the first period and capital goods[8] delivered at the beginning of the second one. While, in the first period, the prices of consumption goods delivered in the second period and coming from processes (c) must be considered as *expected prices*, since it is not possible to close a deal involving such goods, these price expectations, however, are not exogenously given, as in the temporary equilibrium, but are endogenously determined beside the current prices by a system of equilibrium conditions formally equivalent to that of Arrow–Debreu. In other words, the expected prices of future commodities are exactly those that would have resulted if there were Arrow–Debreu complete future markets.

Moreover, the first-period equilibrium will also determine the quantities of capital goods coming from processes (b). Therefore, the second-period endowments will not be arbitrarily given, but determined by (past) equilibrium decisions, so that, generically, no second-period input will be in excess supply. In this case, Mandler proves the

[7] Since there is no future after the second period, at the equilibrium no capital goods will instead be produced.

[8] Although this will be clarified later, capital good prices, which are determined by the first-period equilibrium, must be interpreted as 'supply prices'; while capital good 'demand prices' are those that will be determined by the second-period equilibrium (cf. below, footnotes 17 and 21).

indeterminacy of the second-period equilibrium under the following conditions: (i) the inputs (or at least some of them) are inelastically supplied; (ii) there are no differentiable production functions; (iii) the number of outputs (consumption goods) is smaller than the number of inputs (different kinds of labour and capital goods).

The proof of this result can be intuitively outlined in the following way (see the Appendix for further details). Let us consider an atemporal equilibrium model satisfying the conditions (i), (ii) and (iii) described above. If input endowments are arbitrarily given, then, generically, there will be some input in excess supply,[9] and so their equilibrium prices will be zero. This is exactly what is needed in order to obtain the neoclassical determinacy result for the case under consideration.

The latter claim can easily be verified by means of a very simple example. Let us consider an economy with a single consumption good and two pure inputs. Given the method of production, a_1 units of input 1 and a_2 units of input 2 are required in order to produce one unit of consumption good. Now, if the input endowments (ω_1, ω_2) are arbitrarily given, we will generically have $\omega_1/\omega_2 \neq a_1/a_2$, so that one of the two inputs, in equilibrium, will be in excess supply. For example, if $\omega_1/\omega_2 > a_1/a_2$, then input 1 will be in excess supply and its equilibrium price will be zero; while the price of input 2 in terms of consumption goods (which is our numeraire) will be $p_2 = 1/a_2$. But, in the very special case in which $\omega_1/\omega_2 = a_1/a_2$, both inputs will be fully employed and every input price vector $(p_1, p_2) \in R^2_+$ such that $1 = a_1p_1 + a_2p_2$ is an equilibrium.

Now, Mandler's second period equilibrium is, in effect, an atemporal equilibrium the initial endowments of which, instead of being arbitrarily given, are such that no input will be in excess supply even though there are linear production activities, the inputs are inelastically supplied and the number of inputs exceeds that of outputs.[10] Mandler's result is, therefore, perfectly consistent with the typical neoclassical indeterminacy argument and, in this respect, is not particularly surprising. However, it deserves closer examination, especially in order to shed light on its economic causes—rather than on the mathematical causes of indeterminacy already explored by Mandler—and on its real significance. This is the aim of the following section, while in Section 4 we will come back to the general question of Sraffian indeterminacy.

3. Sequential equilibria and the tendency to a uniform rate of return on capital

In the Arrow–Debreu intertemporal theory, initial endowments can, and in general must, include different kinds of capital goods, but, as is known, these initially available capital goods are treated as if they were natural resources:[11] the decisions to make them were taken in the past, and there is no memory of their production costs.[12] The case of capital goods that are not initially available is instead different: their quantities will be determined on the

[9] The number of inputs in excess supply will almost always be equal to the difference between the number of inputs and that of outputs.

[10] See, for example, equilibrium system (A11)-(A14) in the Appendix.

[11] As Malinvaud (1953, p. 235, n. 4) writes, in an intertemporal framework, '[t]he distinction between natural resources and produced means of production is not important as far as past activity is concerned'.

[12] For this reason, while in Walras, taking as given the endowments of specific capital goods conflicted with the assumption of a uniform rate of return on their supply prices (cf. Garegnani, 1990B), these endowments create no problems in Arrow–Debreu intertemporal theory, because here there is not a trace of their supply prices.

basis of their supply and demand prices by a mechanism similar to that which in the traditional neoclassical theory leads to a uniform rate of profits.

More precisely, in the intertemporal model, capital goods are produced in the period t only if there are agents that are willing to save a share of their income from that period. By saving, agents do not acquire any direct utility but buy capital goods which they lend to the firms in order to increase their future incomes (and thus consumption). Assuming, for the sake of simplicity, that only circulating capital exists, at t savers pay for capital goods at their supply prices—that is, they cover their cost of production in terms of commodities delivered in period t; while firms take in and use those goods from savers in exchange for a promise of commodities delivered in period $t+1$. The quantity of commodities paid by the firms in $t+1$ for using a capital good represents its demand price.[13]

In equilibrium, the ratio between demand and supply prices must be the same for all the capital goods delivered at time t in *positive quantities*. For example, if the supply prices of the capital goods are expressed in corn delivered at time t, while their demand prices are expressed in corn delivered at time $t+1$, then the capital goods produced will have to have a ratio of demand price/supply price equal to one plus the corn's own rate of interest.[14] In fact, as capital goods are *perfect substitutes* for the saver, if this ratio is greater for some capital goods than for others, savers would buy only the former ones, which would thus be the only produced (or reproduced) capital goods. Hence, the physical composition of capital employed in period $t+1$ will be the result of the tendency to a uniform rate of return on the supply prices of capital goods, which underlies the desire of individuals to maximise the return on their savings.

Having clarified this point and coming back to Mandler's indeterminacy, in his first-period equilibrium, as we saw in Section 2, future price expectations are determined as if there were Arrow–Debreu complete markets. Consequently, the quantities of capital goods that are produced in the first period and are delivered at the beginning of the second will be those which would be determined in the Arrow–Debreu equilibrium. In other terms, the physical composition of capital that is available at the beginning of the second period will be that which in Arrow–Debreu assures the uniformity of the rate of return on the supply price of the capital goods. However, in Mandler's world the markets are incomplete and the demand prices of capital goods, which have influenced the composition of capital at the beginning of the second period, are expected prices, which might not be equal to the prices then determined in the second period equilibrium. It is in fact only at the beginning of the second period that transactions determining these prices in terms of consumption goods delivered in period two will actually take place. And at this point the agents, who had already transformed their saving into capital goods of a certain kind, will be unable to change the decisions taken in the past—that is at the beginning of the first period, whatever the second period prices are.

Thus, taking the argument put forward towards the end of Section 2 into account, if the number of consumption goods produced and delivered in period two happens to be less than the number of inputs, in general there will be a continuum of demand price systems of capital

[13] Here no distinction is made between the price of the service of the capital good and its demand price, since present prices, which are already discounted, are considered in Arrow–Debreu theory. In the case of circulating capital, the undiscounted demand price of capital goods would, of course, be equal to the price of its servicing.

[14] If we argued in terms of discounted prices—namely, if also demand prices were expressed in corn delivered at time t—the capital goods produced would instead have had a ratio of demand price/supply price equal to one. This condition, within the equilibrium system discussed in the Appendix, is represented formally by equation (A9).

goods compatible with the equilibrium. Of these, only the one equal to the prices expected in the first period would guarantee the uniformity of the effective rates of return.[15]

Now, the emerging of new equilibrium price systems in the second period that would be incompatible with the intertemporal equilibrium over two periods, is a direct consequence of the fact that in Mandler's second period equilibrium, capital goods have already been produced and thus capital composition can no longer be changed. Therefore, those price systems that do not allow the uniformity of the effective rates of return now appear as possible equilibria of the second-period economy, thus generating indeterminacy.[16]

Summing up, in Mandler's construction, while the physical composition of capital of the second period economy is determined by the tendency to uniformity of the expected rate of return of capital goods, this tendency does not contribute, in the second period, to determining the demand prices of capital goods. It is thus not surprising that prices are found to be indeterminate: a relevant part of the mechanism contributing to their determination has been thrown out, since the reopening of the markets in the second period has broken the link between demand and supply prices of the capital goods produced. Thus, in the second period equilibrium, the now available capital goods are treated as natural resources pruned of a cost of production and therefore of a supply price, precisely as in the case of the endowments of capital goods of the first period according to the logic of the intertemporal equilibrium theory. However, while true natural resources are given arbitrarily and can therefore generate excesses of supply and hence zero prices, the quantities of capital goods available at the beginning of the second period are not arbitrarily given, but are those 'adjusted' on the grounds of choices made in the first period. In this sense, in Mandler's model we find an *ad hoc* use of the tendency of the uniformity of the rates of return of the supply prices of capital goods: it is used to determine quantities but not prices.[17]

To conclude we would like to stress an implication of Mandler's argument. As shown in Section 2, in his model we find both the assumption of incompleteness of future markets and that of the markets reopening at the beginning of the second period. The former assumption implies that the consumption goods to be delivered in the second period cannot be the object of exchange in the first period, so that their prices are, in the economy of the first period, expected prices—though equal to those to be had if there were Arrow–Debreu complete markets.

Now, thanks to Mandler we know that, even if the equilibrium is unique in the first period, the second period equilibrium would generically be indeterminate. Indeed it is the factor prices that are indeterminate, and consequently those of the consumption goods in terms of one of the factors of production. For example, if food and cloth are produced with labour, carbon and iron, even if the price of food in terms of cloth is equal to the expected one, the price of food in terms of labour or of iron will be indeterminate, and thus generally different from the expected one. This means that future price expectations, which are formulated in the first period by the agents as if they were in an Arrow–Debreu model, will generally be wrong. This result, quite surprisingly, was not noted by Mandler.

[15] The expression 'effective rates of return' is proposed here in contrast with that of 'expected rates of return'.

[16] We formally show this in the Appendix.

[17] As we will argue in the Appendix, if in the second period there is a memory of supply prices of capital goods, so that their demand prices would be determined on the base of the tendency to a uniform rate of returns, it can be demonstrated that (i) indeterminacy disappears and (ii) prices will be determined independently of demand functions.

The heroic assumption of market completeness is commonly justified by arguing it is equivalent to that of agents having complete information and not making systematic forecast errors about future prices, namely by the equivalence between the assumption of perfect foresight and that of Arrow–Debreu completeness of markets.[18] But, according to the above result, such equivalence cannot in fact be advanced, since the second-period commodity prices determined, at the beginning of the first period, on the basis of a system equivalent to that of Arrow–Debreu will be only one of the infinite price systems that will emerge as an equilibrium of the second-period economy. In particular, the saver, who in the first period bought capital goods of a certain kind, will in general receive quantities of goods delivered in the second period different from those expected. And this will be due not to the occurrence of some unpredictable accident but to the fact that expectations formed on the ground of an Arrow–Debreu equilibrium system can account for only one of the infinite configurations that prices can assume in the second period equilibrium.

It follows that (i) contrary to the common opinion, the formal Arrow–Debreu apparatus is not open to a dual interpretation, since it is able to determine the correct future prices only if the markets are truly complete; (ii) hence, the assumption that agents' expectations are formed as if they were in an Arrow–Debreu system amounts to assuming arbitrary expectations, since in both cases you will probably come up with incorrect forecasts. The Arrow–Debreu theory would thus be confined to mere theoretical curiosity regarding an imaginary world where complete future markets exist.

4. Sraffa and the reappraisal of the classical theory of distribution

Something of the above seems to have been perceived by Mandler when he observed that 'there is no mechanism to lead second-period markets to equilibrate at the continuation equilibrium prices' (Mandler, 2003, p. 354) since:

> only the expectations that agents formed during the first period, and not any feature of markets narrowly construed, distinguish the continuation equilibrium from the rest

and

> since agents will foresee this difficulty in the first period, they *will not anticipate any price vector with certainty.* (Mandler, 2003, p. 354, our emphasis).

Consequently, according to Mandler, only two routes would be open for solving price indeterminacy. The first is a sort of a 'Sraffian route' based on factor owners not taking prices parametrically (cf. Mandler, 2003, pp. 341, 354). The second is that of referring to Hicks' (1939) temporary equilibrium with exogenous price expectations, which Hicks himself, however, eventually criticised (cf. for example Hicks, 1965, pp. 73–75). While in fact in this case the revelation of wrong expectations will avoid indeterminacy by leading to unadjusted capital compositions, the equilibrium path of the economy would be determined by *ad hoc* or subjective specifications of the formation and content of beliefs about future labour and land endowments, preferences and techniques, natural and political events, and so on.[19] Therefore, even if the adjustment between demand and

[18] It is usually claimed that sequential equilibrium with rational expectations and perfect knowledge is equivalent to the Arrow–Debreu model in which there are complete markets open only at the beginning of all the times considered. On this point, cf., among others, Grandmont (1977, p. 535) and Lucas (1988, p. 9).

[19] Note that Hicks (1939, p. 204) considered as relevant in the formation of expectations the weather, political news, people's state of wealth, psychology, and so on—factors disregarded in Grandmont (1988) and other recent specifications of the expectation function shaping temporary equilibrium.

supply were in any period instantaneous and thus able to be realised before the fast changes in the data determining the temporary equilibria, these would in any case be arbitrary (cf. Duffie and Sonneschein, 1989) and with no connection to the actual path of the economy.

Alternatively, in order to overcome these and other difficulties (cf. Garegnani, 1990B, 2003; Petri, 2004) of the neo-Walrasian general equilibrium theories, we could come back to those 'normal positions' of the economy, which the first versions of the marginalist theory, from Walras to Wicksell, unsuccessfully tried to determine, but which can be dealt with following Sraffa's revival of the surplus approach to value and distribution. This route, which Mandler refutes, brings us back to some methodological questions, and precisely to that of the existence of Sraffian indeterminacy. It also leads us to answer Mandler's criticisms of Sraffa which we summed up in Section 2.

Let us first consider the alternative to which Mandler leads us. As pointed out by Garegnani (2005A, p. 424), Mandler should recognise that also the initial endowments of capital goods of the intertemporal equilibrium should not be given arbitrarily. In fact, if, according to Mandler, in order to have a 'true' equilibrium, prices operated by avoiding reproducible factors being in excess of supply (cf. Mandler, 1999A, p. 425), this would happen also with respect to initial endowments of his 'first period economy'. Here the stocks of various capital goods, which had been produced in the past and had a cost of production, should not be given arbitrarily (cf. also Schefold, 1985, p. 142), but be endogenously determined in order to avoid zero prices for their services.

In this case—leaving aside stationary states that are far from actually being realised—the neoclassical theory of income distribution needs to take the value of capital per worker available as a given magnitude, as well as in Böhm–Bawerk's, Clark's and Wicksell's attempts. But it is precisely the necessity to treat capital as a value magnitude able 'to change its form' (cf. Wicksell, 1934, p. 192)[20] that led to the failure of the first versions of the marginalist theory, since that magnitude cannot, as Mandler recognises, be taken as given before distribution is determined, nor can it be put into the production function, as Wicksell himself showed.

It can thus be said that Mandler's analysis ends up by justifying Sraffa's effort to discover a theory of value alternative to the marginalist one, and through it to help respond to the idea of indeterminacy itself in Sraffa's price system.

When assuming constant returns to scale, the price system equation (1) of Section 2 coincides with that detectable, for instance, in Walras.[21] It actually tells us only that commodity prices must be equal to the normal costs of production. And as long as wages, and thus costs, are not determined, the system has a degree of freedom that is precisely what permits prices to change parametrically.

The secret of Sraffa's missing equation (cf. Mandler, 2008), and therefore the difference between Sraffa and Walras, thus resides in what they consider eliminates that degree of

[20] In this respect it is worth noting that, when analysing the debate between Hicks and Robertson concerning the possibility of tracing meaningful curves for the marginal products, Mandler (1999B, pp. 18, 30) admits that we should consider capital as a value magnitude able to change its concrete form, having otherwise (cf. also Hicks, 1932; Marshall, 1980, p. 516; Robertson, 1957–58, p. 27) a meaningless short-period marginal product. It amounts to admitting that these curves should not be traced.

[21] Assuming overabundant land and circulating capital, in Walras the supply price P_k of the capital good k produced by the quantity k_p of labour, the quantity k_k of itself and the quantity $k_{k'}$ of another capital good, is: $P_k = k_p v_p + k_k v_k + k_{k'} v_{k'}$ where v_i (for i = p; k; k') are respectively the service prices of labour and of the two capital goods k and k'. Denoting as K the output of commodity k, and putting $k_p = L_k/K = l_k$; $v_p = w$; $k_k = K_k/K$ e $k_{k'} = A_k/K = a_k$, since, according to Walras, the demand price of the capital goods must be equal to their supply prices—that is (in the case of circulating capital) $P_j = P_j^d = v_j/(1+r)$, where r is the rate of interest—we will obtain, by substituting, Sraffa's price equation of commodity K.

freedom, that is in their theories of distribution (cf. also Petri, 2003). While for Walras it depends on demand and supply on the basis of the substitution principle among factors and goods—which leads him to close the system (1) by adding the conditions of equality of demand and supply for goods and factors of production (cf. Walras, 1874[1954], pp. 426–27, 493); Sraffa rediscovers the surplus approach of the classical economists and Marx. He thus determines relative prices taking wages as given and obtaining profits as a residuum, given gross outputs (cf. Sraffa, 1960, ch. v) and, at least in the first part of *Production of Commodities*, the methods of production. As is now emerging from an analysis of his unpublished manuscripts (see, e.g., Garegnani, 2005B; Gerhke and Kurz, 2006), Sraffa's rediscovery of the surplus approach of Petty, Smith and Ricardo (cf. Marx, 1954–62, Vol. I, pp. 80–81, n. 1, 2) passed from the initial critique of Marshall's real costs to the refutation of any Marshallian interpretation of Ricardo. And although with differences and contradictions among the classical economists themselves (see, e.g., Marx, 1978, Vol. I, pp. 61, 81–82, 87–88), which Sraffa contributed to clarifying with his work on Ricardo's writings, this approach, unlike that of Walras, does not put distribution on the same footing as the process of price determination, but views the former as the result of social conditions that are more fundamental than those determining relative prices (cf. Dobb, 1973)—thus taking as given the wage rate in *that* determination. As pointed out by Marx (1954–62, Vol. III, p. 837), 'in every country, at a given time, this regulating average wage is a given quantity. The value of all other revenue thus has its limit'.

But as stated in Section 2, Mandler's converse view of Sraffa as a special case of the neoclassical theory arises from the idea that a meaningful competitive equilibrium would need the market-clearing conditions to be satisfied in all markets. According to Mandler, Sraffa was ambiguous in this respect, especially having left 'the situation for labor (...) unspecified' (cf. Mandler, 2008). We thus have the alternative, according to Mandler, either to admit the possibility of wages falling to zero, but thus failing to have a significant economic theory, or to introduce the substitution principle, which is precisely what happened, Mandler maintains, after the appearance of Thornton's argument of price indeterminacy (cf. Mandler, 1999B, pp. 55, 64).[22]

Undoubtedly this is the central argument underpinning Mandler's way of interpreting Sraffa. However, the choice he leaves us merely reflects his unwillingness to accept the validity of a theory of value different from the neoclassical one (cf., e.g., Mandler, 1999B, pp. 54, 71–73), where no relative scarcity principle for labour and capital is present and prices have only the role of redistributing the surplus among sectors in order to assure a uniform rate of profits. It is the same unwillingness that leads Mandler to view in terms of indeterminacy the idea, taken up by Sraffa (cf. Pivetti, 2000), of income distribution as not predetermined by natural or technical factors, but depending on the workers' relative bargaining power—forgetting that, in the classical theory, what shapes that relative power, and hence distribution, is an essential part of the theory.

[22] Note that, according to Mandler, it is Thornton's critique of J. S. Mill that led to the abandonment of the classical theory. As for Arrow (1968[1983], p. 109), for Mandler it was in fact the limits of the population principle that kept wages at the subsistence level, on the one hand, and Thornton, Longe and Walker's critique of Mill's wage fund doctrine, on the other, which required introducing the substitution principle to determine the wage rate. However, Mandler erroneously ascribes (cf. Stirati, 1999) to Smith and Ricardo the wage fund doctrine. Moreover, in discussing the classical wage theory, he does not see that it is implicit in an historically determined notion of subsistence that wages can remain above subsistence for a long period of time (cf., e.g., Smith, 1776[1976], I. viii, 11–18, pp.83–86), in accordance with the 'relative advantage' of the parties in wage bargaining.

Let us try to clarify. As stressed by Garegnani (2007), the surplus approach of Smith and Ricardo proceeds through separate, though connected, logical stages. Given the wage rate, the normal outputs, and the methods of production, classical economists determine—as in Sraffa (1960)—relative prices and the profit rate, thanks to univocal and exact relations, which originate from the tendency of prices toward normal costs of production under the pressure of competition. The determination of those data—and thus the analysis of the interactions among them and with relative prices—is carried on in a distinct and more concrete stage of analysis, where the influences of social factors and their historical specific determinations are fully considered. Examples of this kind of analysis are Smith's chapters of *The Wealth of Nations* on the division of labour, or Smith's and Marx's wage theories, or further, theirs and Ricardo's scenarios of the tendencies of capital accumulation—which were advanced with the help of specific hypotheses and of those economic laws obtained in the more abstract level of analysis. Thus, according to the classical economists, wages will depend on the labour market conditions (that is, on Ricardo's proportion of labour demand to supply or, in present-day language, on labour unemployment and underemployment), but also on the workers' degree of organisation, and on the social and political situation of a country—all elements that contribute to determining whether wages will be above or equal to the subsistence level inherited from the past standard of life of the labouring class. Likewise, the trend of capital accumulation is seen to influence labour productivity and thus prices and income distribution. Smith thus wrote:

> (t)he increase of demand (. . .) though in the beginning it may sometimes raise the price of goods, never fails to lower it in the long run. It encourages production, and thereby increases the competition of the producers, who, in order to undersell one another, have recourse to new division of labour and new improvements of art, which might never otherwise have been thought of (. . .). (Smith, 1776[1976], V. i. e, 26, p.748).

It is precisely his reluctance to accept this composite method of analysis that led Mandler (1999B, p. 69) to criticise the absence in Sraffa of a simultaneous determination of outputs and prices. But, first, it does not mean (cf. Mandler, 1999A, p. 694) that in Sraffa there is no equality between demand and supply of *goods*, or that those outputs are *ex post* quantities. Sraffa in fact deals with what Smith called the natural prices (cf. Sraffa, 1960, p. 9), which are the prices that prevail when supplies equal the 'effectual demands'.[23] Second, the separate determination of output and prices is exactly what theoretically permits the (empirically suggested) extension to the long run, when productive capacity can be adjusted, of the principle of effective demand, as well as the treatment, within economic

[23] According to classical economists, these *natural* prices will be determined by 'the costs of production', and not by 'supply and demand' (see Ricardo, 1951–1973, II, pp. 53, 225, 266), while the idea of an equality between supply and demand with respect to the *market* prices will appear only with J. S. Mill (cf. Bharadwaj, 1978; de Vivo, 1981). Indeed, according to Smith (1776[1976], I. vii, 10, p.74, our emphasis), if the supply happens to be greater than the effectual demand, '[s]ome part [of the quantities brought to market] must be sold to those who are willing to pay less, and the low price which they give for it must reduce the price of the whole', but '(t)he market price will sink *more or less* below the natural price, according as the greatness of the excess increases more or less the competition of the sellers, or according as it happens to be more or less important to them *to get immediately rid* of the commodity'. Cf. also Ricardo (1951–1973, I, XXI, p. 298), who stated that a manufacturer can be 'unwilling to sell [finished goods] at very depressed prices'.

theory, of the social nature of consumption.[24] Finally, that simultaneous determination is not necessary in Sraffa's classical theory. As shown by the non-substitution theorem, given the wage rate and the methods of production, demand conditions play no role in determining relative prices. And the never neglected interactions between prices and production at the industry level and in the economy as a whole, can (and must) be studied with specific investigations, according to circumstances, in a stage of analysis that is distinct from that in which prices are ultimately fixed.[25]

To return to the meaning of Sraffa's price equations, it follows precisely from such a separate determination of prices and quantities that Smith and Ricardo were able to determine a normal position of the economy with unemployment and positive wages—of which, according to Hahn (1987), economic theory should endow itself, but for which Mandler, as we have seen, reproached Sraffa. On one side, Smith or Ricardo saw free competition only as something pushing toward a uniform rate of profits and uniform wages for the same kind of work, on the basis of the free movement of labour and capital among industries. On the other, as acknowledged by Mandler himself (1999B, p. 54), they had no conception of an automatic tendency toward the full employment of resources. Hence they were naturally led to admit that competition acts only within a set of social norms and institutions, including those assuring (an at least relative) wage inflexibility, and thus an ordered reproduction of the economy (cf. Levrero, 2011)—guaranteeing, as Marshall (1980, p. 578) wrote, that workers will not become 'extravagant from the point of view of society at large'. The idea of wages necessarily falling as long as an excess of labour supply persists is meaningful, in fact, only if such a fall leads to an increase in employment,[26] as in the wage-fund doctrine or in neoclassical theory. It could not thus arise in the classical theory, where it would merely lead to wages approaching zero.

However, Mandler's insistence on introducing 'demand and supply conditions' to determine relative prices also arises from his firm belief in the need for dating prices also as

[24] Cf., e.g., Smith (1776[1976], V. ii. k, 1–7, pp.869–872) on the distinction between the necessaries and the 'luxuries of the poor', or Ricardo's (1951–1973, I, pp. 241 and 343–44) emphasis on the influence of habits on consumption. Conversely Veblen's effects or the dependence of tastes on prices and consumption patterns are treated by the neoclassical theory (cf., e.g., Hammond, 1976; Pollack, 1977) but with *ad hoc* assumptions unable to fully recognise the dependence of wants on production and the social context. The difficulty of dealing with the social dimension of consumption is indeed recognised e.g. by Geanakoplos (1987, p. 117) when observing that neoclassical theory disregards self deception, Freudian split psyche, Odysseus-like changes of heart, thrift behaviour shaped according to one's role in society.

[25] The fact that this is the more advantageous method of studying the relations between prices and quantities in the classical theory originates from two characteristics that make it incompatible with demand *functions* for goods. First, not arising from any idea of price flexibility leading to full employment of resources, in this theory the income of individuals would still be indeterminate even if prices are given. Moreover, even if incomes were determinate for a given price system, the absence of a tendency to full employment would render arbitrary any assumption regarding their changes when prices change. The second characteristic depends on the importance attached to social and historical elements by the classical theory. The consumption habits of social classes in this approach lie at the heart of individual consumption choices. Consequently, even if individual incomes were univocally determined together with the price system, changes in the latter could lead to effectual demands varying mainly by modifying the social structure and the habits of individuals. Yet in this case a change would occur that was so radical that holding the other circumstances unchanged would be unjustified.

[26] In this respect it should be noted that the parallelism put forward by Mandler (2008) between land and labour is incorrect. First, no problem for an orderly reproduction of the economy arises with zero rent on the marginal land. Second, in Ricardo, it is a consequence of the notion itself of free competition leading to a uniform rate of profits and uniform wages. Third, as stated above, in the classical theory, free competition can only drive distribution variables towards their normal level, as it is zero, as in the case of Ricardo's rent for marginal lands, or strictly positive, as the wage rate must be. Of course, such a point must be distinguished from that previously considered with respect to the possible influence of the condition of the labour market in shaping the average or *normal* wage rate.

a consequence of the 'erratic character' of consumers' tastes and of the methods of production (cf. Mandler, 2002, p. 204). This raises two problems already faced in Sraffian literature, and so we shall only mention them in passing. Before proceeding, however, it is worth noting that, when commenting the fact that 'the economic world is subject to continual change (. . .)', Marshall (1980, p. 319, n. 1) observed:

> it is true that we do treat variables provisionally as constants. But it is also true that this is the only method by which science has ever made any great progress in dealing with complex and changeful matter, whether in the physical or moral world.

The first problem raised by Mandler again refers to the well-known statement that, since prices depend on the methods of production, these methods—in the case of non-constant returns to scale—depend on outputs, and the latter (in the shape of demanded quantities) on prices, then outputs and prices must be determined simultaneously. Otherwise we must assume constant cost or zero price elasticity of demand (cf., e.g., Blaug, 1958; Marshall, 1980, pp. 814, 834–5). Now, the point here is precisely that mentioned above, referring to the method of analysis of the classical economists: the interactions among prices and outputs—which the classical economists do not neglect[27]—must be treated, in the classical theory, by an iterative process determining the data shaping the normal position of the economy, and by a comparison of those normal positions (see Garegnani, 1990A; Schefold, 1990)—as performed for instance by Ricardo when analysing the effects of an increasing population on the rate of profits in the case of decreasing returns in agriculture. It also should be noted that in this respect the relevant output changes influencing techniques are those associated with capital accumulation, and that also in this case the methods of production (and thus prices) can vary discontinuously (cf. Sraffa, 1960, p. 77). Moreover, it seems useful to distinguish here between changes in the subjective conditions of production (for which no price variation will occur, and the firms first adopting the more advanced techniques will only obtain extra profits), and changes in the social conditions of production, which will determine a change in distribution and are not usually so rapid as to upset any rational calculation of investors with respect to the normal methods of production (cf., e.g., Hicks, 1965, pp. 295–96; Smith, 1776[1976], II, III, section 32).[28]

The second problem involves instead facing the more general question of what notion of equilibrium we find in the classical theory and in Sraffa, and of how to deal meaningfully with economic changes. In this respect we briefly stress only four elements that Mandler seems to have overlooked, indicating them by letters for the sake of simplicity:

(A) Following Hicks (1939, pp. 117–19), Mandler erroneously identifies the normal positions of the economy with the stationary states as conceived nowadays. Although Smith and Ricardo—as well as Walras, Marshall and Wicksell—viewed those positions

[27] The *normal* effectual demand of Smith (1776[1976], I, VII), which is a single quantity, can of course vary when prices change, but not according to any univocal relation, as stated in note 25. The same can be said with respect to the *actual* effectual demands (those differing from the *average* annual consumption of the commodities: cf. Ciccone, 1999). In this respect, those who, like Rankin (1980, p. 247), discern a unique given elastic demand function, overlook the fact that no general relation between market prices and demand can be advanced when considering expectations regarding future supplies and prices, the opinions regarding the temporary or permanent nature of discrepancies between supplies and normal demands, rationing, speculations on stocks, changes in the market prices of labour and capital, the ability and willingness to pay when prices are different to the natural ones, and so on.

[28] Further, changes in the social conditions of production could indeed favour the process of adjustment to a normal position, by facilitating and accelerating the circulation of knowledge, or by rendering the production plants more flexible (cf. Parrinello, 1990).

as the 'centres of gravity' of actual prices under the pressure of competition, i.e. as a 'rest position' of the competitive process, this does not imply that normal prices must be constant over time. It is in fact the relative persistence (and not the constancy) of their determinants that makes it possible to argue as if they remained unchanged (cf. Marx, 1954–62, Vol. II, pp. 393–95; Marshall, 1980, pp. 206–7, V, v, 3; Wicksell, 1934, pp. 154–5) and, in so doing, to isolate the (more or less) persistent causes of value from the accidental and transitory ones,[29] without, however, sterilising any tendency to change as in the modern stationary state. As asserted by von Mises (1933, p. 117), who labelled this method as static, it is precisely the possibility of thus referring to average or normal values that allows the effects of changes in the determinants of normal prices to be studied:

> One must not commit the error of believing that the static method can only be used to explain the stationary state of an economy, which, by the way, does not and never can exist in real life; and that the moving and changing economy can only be dealt with in terms of a dynamic theory. The static method is a method which is aimed at studying changes (...).[30]

(B) Mandler does not consider that Sraffa is not obliged to abandon that method of normal positions of the economy, which, until some decades ago, was viewed as the only one capable of assuring that the prices determined by economic theory could be conceived as those towards which actual or market prices will tend (on average) with the repetition of transactions.[31] In particular, in order to determine distribution, classical theory does not need either a given capital in value or an inverse relationship between the employed capital and the rate of interest. Consequently, it does not address the problems regarding capital as a factor of production, which starting with Hicks (1939), led (cf. Garegnani, 1976; Milgate, 1979) to distinguish prices and commodities by dates, and to include in the actual definition of equilibrium the price changes that occur (or are expected to occur) in a non-stationary economy.

(C) It is not clear why price dating would facilitate a true dynamic analysis. First, there is no clear idea of the time horizon to be considered, or how to forecast the tastes of the still unborn generations, or future changes in the methods of production (cf. also Hayek, 1935, p. 251). Not surprisingly, in the intertemporal general equilibrium it is assumed that, oddly enough, the entrepreneur 'knows now what input–output combinations will be possible in the future' although 'he may not know now the details of the technical processes which will make them possible', or it is assumed that 'preference orderings and production sets' are 'known to the actors in the present and

[29] A similar notion of equilibrium can easily be found in natural sciences. Cf. in particular that of quasi-stationary equilibrium in thermodynamics or in the kinetics of enzyme action.

[30] It is quoted and translated by Kurz and Salvadori (1998, p. 21). For instance, it is the comparison of normal positions that allowed Wicksell to highlight the effects of capital accumulation on distribution. This would have been impossible if he were comparing two modern stationary states, since, except in the case of the multiplicity of equilibria, it is inconceivable that these differ only in the amount of capital in use. On the presence of different notions of stationary state in economics cf., on the other hand, Robbins (1930) and Frisch (1936, pp. 101, 103).

[31] Within the traditional neoclassical framework, this tendency was formulated in terms of stability. But with the neo-Walrasian intertemporal versions, the conception of stability itself seems to involve new problems. In particular, as Malinvaud pointed out: '[t]here might be some difficulty with stability since time must enter both the definition of equilibrium, and the process of convergence towards equilibrium. I am not aware that anyone has really looked into the matter. But it seems to me that the problem is just to find a proper definition of stability' (Malinvaud, 1961, p. 152).

in the future' (cf. Debreu, 1959, respectively, pp. 38, 56–57)—which implies excluding, for instance, the effects of newly invented goods on tastes, as well as technical progress, as they are unpredictable by definition. Second, in order to study the tendencies of the economic system, we should actually refer to what Hicks (1965) called 'exercises of comparative dynamics', that is to a comparison of the changes occurring in outputs and dated prices when a change in the data of the initial path occurs. Therefore, by itself, intertemporal equilibrium is no help to us in studying change. On the contrary, its above-mentioned arbitrary assumptions can limit such an analysis, as on average that equilibrium has no relation to the actual path of the economy, and it obscures phenomena of path dependence, or reciprocal influences between distribution, technical progress and capital accumulation;

(D) When considering the classical theory—in which, as we have seen, there is no intrinsic need to date prices—the route reappraised by Mandler for studying changes thus appears sterile compared with the method of normal positions used by Ricardo or Marx when, for instance, they propose scenarios of growth based on plausible guesswork regarding the speed of technical progress or capital and population increases. On the basis of intertemporal equilibria we could only emphasise the formal aspects of modelling and lost link between theory and the real world, as has already been pointed out with respect to neoclassical theory (cf. Clower, 1995; Backhouse, 1998; Blaug, 2003). This is the same peril arising from formalism, which von Neumann saw with respect to mathematics itself:

> As a mathematical discipline travels far from its empirical source (. . .) it becomes more and more purely aestheticizing, more and more purely l'art pour l'art. This need not be bad, if the field is surrounded by correlated subjects which still have closer empirical connections (. . .). But there is a grave danger that the subject will develop along the line of least resistance, that the stream, so far from its source, will separate into a multitude of insignificant branches (. . .). (von Neumann, 1947, p. 196).

Summing up, Mandler is wrong both when pointing out that we need the conditions of demand and supply in order to determine relative prices and when interpreting the degree of freedom in Sraffa's price system in terms of indeterminacy. Indeed, when Sraffa's theory is placed within the classical approach to value and distribution that he explicitly refers to, no price indeterminacy actually arises.

Moreover, Mandler's sequential indeterminacy appears to be neither necessary nor opportune in order to put forward an explanation of distribution of the kind we find in Sraffa. As shown in Section 3, in this respect, Mandler combines elements of intertemporal equilibrium with others alien to and incompatible with that approach. Sequential indeterminacy arises in fact because Mandler determines the physical composition of capital on the basis of the tendency to uniformity of the rates of return of capital goods on their supply prices, but without allowing this tendency to contribute to the determination of the demand prices of the same goods.

We are left with Mandler's suggestion that, unlike in an Arrow-Debreu economy, the physical composition of capital must not be arbitrarily given. As we have argued, this brings us back to those normal positions of the economy which the first versions of marginalist theory, from Walras to Wicksell, tried unsuccessfully to determine. But it is also what enables us to account for Sraffa's efforts to put forward a theory of value alternative to the neoclassical one.

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Appendix: sequential equilibrium

Mandler considers a sequential intertemporal economy with two periods: 0–1 and 1–2. In the first period we have a temporary equilibrium, which is determined at instant 0, and in which expectations are determined as if there were Arrow–Debreu complete markets. Conversely, in the second period, we have an atemporal equilibrium with production, determined at instant 1, in which the initial endowments of capital goods are not given arbitrarily, but are those arising from the decisions taken in the previous period.

The equilibrium of the first period is formally equivalent to an Arrow–Debreu model, with some variants, which we will describe in detail below. For the sake of simplicity we consider a model with only five commodities in any period, namely two pure consumption goods (commodities 1 and 2), and three factors of production (commodities 3, 4 and 5). Commodity 5 is the non-reproducible factor, labour, while commodities 3 and 4 are circulating capital. The price $p_{i,j}$ is that of commodity i delivered at time j, and the price $p^e_{i,1}$ indicates an expected price. Similar notation is used for outputs $y_{i,j}$ and $y^e_{i,1}$, for the initial available quantities of factors $\omega_{i,j}$ and for the demand functions of the consumption goods $z_{i,j}$. With respect to the technology, $a_{i,j}$ is the quantity of commodity j required to produce one unit of commodity i. In its extensive form, the equilibrium system of the first period is

$$z_{i,0}\left(p_{1,0},\ldots,p_{5,0},p^e_{1,1},\ldots,p^e_{5,1}\right)=y_{i,0} \tag{A1}$$

for i = 1, 2

$$z_{i,1}\left(p_{1,0},\ldots,p_{5,0},p^e_{1,1},\ldots,p^e_{5,1}\right)=y^e_{i,1} \tag{A2}$$

for i = 1, 2

$$\omega_{i,0}\geq a_{1i}y_{1,0}+a_{2i}y_{2,0}+a_{3i}y_{3,1}+a_{4i}y_{4,1} \tag{A3}$$

with "=" if $p_{i,0}>0$ for i = 3, 4, 5

$$y_{i,1}\geq a_{1i}y^e_{1,1}+a_{2i}y^e_{2,1} \tag{A4}$$

with "=" if $p^e_{i,1}>0$ for i = 3, 4

$$\omega_{5,1}\geq a_{15}y^e_{1,1}+a_{25}y^e_{2,1} \tag{A5}$$

with "=" if $p^e_{5,1}>0$

$$p_{i,0}\leq a_{i3}p_{3,0}+a_{i4}p_{4,0}+a_{i5}p_{5,0} \tag{A6}$$

with "=" if $y_{i,0}>0$ for i = 1, 2

$$p_{i,1}=a_{i3}p_{3,0}+a_{i4}p_{4,0}+a_{i5}p_{5,0} \tag{A7}$$

for i = 3, 4

$$p^e_{i,1}\leq a_{i3}p^e_{3,1}+a_{i4}p^e_{4,1}+a_{i5}p^e_{5,1} \tag{A8}$$

with "=" if $y^e_{i,1}>0$ for i = 1, 2

$$p^e_{i,1}\leq p_{i,1} \tag{A9}$$

with "=" if $y_{i,1}>0$ for i = 3, 4

$$p^e_{1,1}=1 \tag{A10}$$

For Walras's law, of the 19 equations of the system only 18 are independent. Also the unknowns number 18: six outputs (four effective and two expected), and 12 prices, that is the five prices of the commodities delivered at time 0, the two supply prices of the capital goods $p_{3,1}$ and $p_{4,1}$, and five expected prices.

Unlike the Arrow–Debreu model, where they usually do not appear explicitly, through the conditions of equation (A7) we have introduced the supply prices of capital goods, which are those paid in the first period by the savers when they buy the capital goods. Prices $p^e_{3,1}$ and $p^e_{4,1}$ are instead the demand prices of the capital goods, that is the prices which savers expect to realise in the second period when they give the goods to the firms. The conditions of equation (A9) impose equality between the supply and demand prices of the capital goods, which in equilibrium are produced in strictly positive quantities. If we substitute the conditions of equation (A7) into equation (A9), the system would be written in the conventional way.

By solving system (A1)-(A10) we will determine the quantities of capital goods produced in the first period and delivered at the beginning of the second period: $y_{3,1}$ and $y_{4,1}$. Let us thus put $y_{3,1}=\omega_{3,1}$ and $y_{4,1}=\omega_{4,1}$. Given these quantities and the available amount of the non reproducible factor $\omega_{5,1}$, in the equilibrium of the second period we will have to determine the seven variables that were expected in the first period, that is two quantities and five prices. We will have:

$$z_{i,1}\left(p_{1,1},\ldots,p_{5,1}\right)=y_{i,1} \tag{A11}$$

for i = 1, 2

$$\omega_{i,1}\geq a_{1i}y_{1,1}+a_{2i}y_{2,1} \tag{A12}$$

with "=" if $p_{i,1}>0$ for i = 3, 4, 5

$$p_{i,1}\leq a_{i3}p_{3,1}+a_{i4}p_{4,1}+a_{i5}p_{5,1} \tag{A13}$$

with "=" if $y_{i,1}>0$ for i = 1, 2

$$p_{1,1}=1. \tag{A14}$$

System (A11)–(A14) is an atemporal equilibrium where the endowments of capital goods $\omega_{3,1}$ and $\omega_{4,1}$ are, however, not arbitrarily given, but those determined by the system (A1)–(A10). Thus, if the endowment of labour $\omega_{5,1}$ is such that the system (A1)–(A10) determines an expected strictly positive wage rate $p^{e}_{5,1}$, then the equations (A12) will all be satisfied with equality at the equilibrium, so that we will have sequential indeterminacy. In fact, since the outputs of capital goods $y_{3,1}=\omega_{3,1}$ and $y_{4,1}=\omega_{4,1}$ are both strictly positive, for equations (A9) the expected demand price, at the equilibrium of the first period, must have been equal to the supply price of both the capital goods. Consequently, since the supply prices are certainly greater than zero (otherwise all the prices would be zero), then the equilibrium conditions (A4) of the first period must both be satisfied with equality, that is capital goods are produced in amounts such that, in the second period, there are no excesses of supply. Moreover, except in the case of an excessive amount of available labour $\omega_{5,1}$, the quantities of capital goods produced will be those guaranteeing full employment (otherwise, distribution would change in a way that the relative price of the consumption good requiring relatively more labour to be produced would fall, thus raising the demand for labour). Finally, if conditions (A4) and (A5) of the equilibrium of the first period are satisfied with equality, then there is at least one price system such that also the corresponding conditions (A12) of the equilibrium of the second period will be satisfied with equality.

You may wonder why in the equilibrium of the second period an indeterminacy emerges that is absent in that of the first period, since the conditions of the former equilibrium are a subset of those of the latter. In particular, in numerical order, they are the same as the conditions (A2), (A4), (A5), (A8) and (A10).

The answer resides both in the conditions of equation (A7), which define the supply prices, and those of equation (A9), which impose the equality between the supply and demand prices of the capital goods produced in strictly positive amounts. These conditions are those that in the equilibrium of the first period assure that there will not be any indeterminacy of the expected prices. Indeed, within the infinite system of expected prices that in the first period satisfy conditions (A2), (A4), (A5), (A8) and (A10), only one (except in the case of multiplicity of equilibria, which we are excluding) is compatible with the others, and in particular with conditions (A7) and (A9). Conversely, in the second period those price systems that are incompatible with the equality of supply and demand prices of produced capital goods again emerge as possible equilibria, since these conditions of equality are absent in the system (A11)–(A14).

By way of further proof, we can assume that in the second period individuals retain a memory of the equilibrium supply prices $\bar{p}_{3,1}$ and $\bar{p}_{4,1}$, which they paid in the first period

to buy the capital goods, so that they will be willing to give these goods to the firms only at those supply prices. In this case, the prices of capital goods will no longer be included among the unknowns of the system (A11)–(A14), but will be given at the values $\bar{p}_{3,1}$ and $\bar{p}_{4,1}$. This evidently means that the tendency to uniformity of the rate of return will determine not only the quantities but also the prices of capital goods. This would avoid indeterminacy, but the equilibrium prices of the second period in general will be determined only by the conditions of equation (A13), namely without any help from 'market clearing' conditions of factors and goods.