A Remark on Intensive Differential Rent and the Labour Theory of Value in Ricardo

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In the *Principles*, Ricardo defended his labour theory of value by arguing that even where all cultivated land pays rent, the last dose of capital employed on the land does not and there is therefore no rent in the price of the product of this capital.

We will show, on the contrary, that the intensive differential rent paid on land of the worst quality under cultivation enters into the agricultural product price and so, even in the most favourable case, commodities are no longer exchanged at prices based on the quantities of labour they embody.

INTRODUCTION

The primary result of Ricardo's analysis is undoubtedly the presence of an inverse relationship between the wage rate and the rate of profit. With the aid of the labour theory of value, Ricardo succeeded where Adam Smith had failed and pinpointed the link between profits and wages.

Although many of Ricardo's conclusions have been shown to be valid in general, his arguments are strongly based on the hypothesis that commodities are exchanged with one other at a ratio determined by the quantities of labour they embody. He was therefore obliged first to deny Smith's claim that prices are not determined by the labour they embody in a developed country and second to refute the objections raised by Say and Malthus.

As is well known, Ricardo's labour theory of value is essentially grounded on two assertions: (i) that the employment of capital in the production of each commodity is proportional to the employment of labour; (ii) that rent is not a component part of the prices of commodities. The first was intended as an approximation, in the sense

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that exceptions were possible and their relevance should be judged case by case. The second is instead a result that Ricardo believed he had established through his theory of differential rent. We shall focus here solely on the latter, as the former has already received a great deal of attention.

If extensive differential rent were the only possible form, then Ricardo's result would hold, as the price of agricultural products would be determined in this case by the cost of production on the least fertile land under cultivation, and land of this quality pays no extensive differential rent at all. In this case, as Ricardo claimed, "rent is not a component part of the price of commodities" (Ricardo, 1951-73: vol. 1, p. 46).

However, although differences in the quality of land always involve extensive differential rent, it can hardly be maintained that the rents of landowners consist exclusively of this element. Where every piece of land is owned, there is clearly no land that is cultivated without the payment of rent to the owner. And this is the objection Malthus and Say raised against Ricardo's theory.

Ricardo did not reject the argument that there is no cultivated land yielding no rent to its owner. He confined himself to defending his theory on the basis of intensive differential rent,[1] arguing that even if Malthus and Say were right to believe that all the cultivated land received rent, the last dose of capital employed on this land (already in cultivation) would still pay no rent and his theory would thus be valid in any case.[2] The ultimate foundation of Ricardo's idea that rent does not enter into the price of commodities therefore appears to be the theory of intensive differential rent.

Ricardo ultimately succeeded in defending his idea and convincing several generations of economists[3] that even though all cultivated land receives rent, the last dose of capital employed on land pays no rent and therefore rent does not enter into the price of commodities.

With the aid of Sraffa's rigorous restatement of differential rent theory,[4] we intend to show here that Ricardo's argument about intensive rent is misleading and the widely accepted conclusion that intensive rent does not enter into the price of agricultural commodities is incorrect.

In particular, after restating Ricardo's argument, we shall analyse intensive differential rent from a different point of view and, using the same data as Ricardo's example, show that it enters into the (relative) price of corn. As a result, when there are intensive rents, commodities cannot be exchanged at prices determined by the embodied quantities of labour.

RICARDO'S ARGUMENT

Ricardo presents his theory of intensive differential rent by means of an example in the second chapter of the *Principles*. He imagines that a capital of £1,000 – which can be regarded as wages anticipated at the beginning of the year – applied on an acre of the most fertile land gives an output of 100 quarters of corn, whereas the same capital applied on an acre of less fertile land would give 80 quarters. He then observes that if the application of a second £1,000 of capital on an acre of the most fertile land gave rise to an increase in product of 85 quarters, this use of capital would be clearly more advantageous, where possible, than the cultivation of less fertile land.

In this case, rent arises even without different qualities of land being cultivated, in that Ricardo views the difference between the 100 quarters obtained by the first dose of capital and the 85 quarters obtained with the second – i.e. 15 quarters, or the value of 15 quarters – as representing the landowner's rent for an acre. The amount of (gross) profit on a capital of £1,000 is thus the equivalent of 85 quarters of corn for the first investment of capital as well as the second. Ricardo concludes that:

> the capital last employed pays no rent. For the greater productive powers of the first £1000, fifteen quarters is paid for rent, for the employment of the second £1,000 no rent whatever is paid. (Ricardo 1951-73: vol. 1, p. 72)

Though capable of showing that more intense cultivation of the most fertile land can be more advantageous than cultivating inferior land, Ricardo's argument in no way proves that rent does not enter into the relative price of corn.

The arbitrary division of capital into doses of £1,000 each is in fact misleading and induces the reader to believe that the last dose of capital employed alone gives an output of 85 quarters of corn. In other words, these 85 quarters seems to be the product of £1,000 capital, without any (further) employment of land. This impression is incorrect, however, because corn cannot be produced without land.

The application of a second dose of capital on the most fertile land derives, as Sraffa (1960) shows, from the possibility of cultivating land of this quality by two alternative methods. With the first, method α, 100 quarters of corn are obtained by employing a capital of £1,000 on an acre of land, which means that one quarter of corn is obtained with a capital of £10 and 0.01 acres of land. With the second, β, 185 quarters are produced by employing a capital of £2,000 on an acre of land, and so the production of one quarter of corn requires a capital of £10.81

and 0.0054 acres of land. Corn is produced by means of capital and land both with method α and with method β.

As a result, when the rent rate is equal to the value of 15 quarters of corn, every single unit of capital employed pays a rent. In particular, every £1 of capital employed pays a rent corresponding to the value of 0.015 quarters of corn with method α and 0.0075 quarters with method β.

With his doses of capital, Ricardo instead seeks to present method β as a combination of method α and a third method making it possible produce corn with no land. The argument is misleading, however, since Ricardo then uses this imaginary third method to prove that rent does not enter into the price of corn.

If there were a method of producing corn without land, it would be a mere truism to assert that rent does not enter into the cost of corn. No such method exists, however, because the investment of the second dose of capital cannot take place independently of the first. This fact becomes clear when we observe that, according to Ricardo's reasoning, a change in the quantity of corn obtained from £1,000 with method α – from the first dose of capital – would entail a change in the amount of product obtained from the last dose of capital.[5]

Whenever rent is paid for the use of land of a certain quality, every single unit of capital invested in that land pays a rent. As a result, if an intensive differential rent is due for the last piece of land under cultivation, then rent enters into the price of corn. This is proved in the next section by means of a simple example based on the same data used here.

A DIFFERENT ARGUMENT

Let us consider an example with only two commodities: an agricultural product, say corn, and an industrial product, say steel.

Let us assume that capital in each sector consists exclusively of wages paid in advance for the period of production, which is one year in both sectors. As a result, capital and profits can clearly be seen to be proportional to the employment of labour in each sector. If rents do not enter into the price of commodities, as Ricardo claims, they should therefore be exchanged at a relative price equal to the ratio of the amounts of labour they embody.

The total quantities to be produced are fixed and equal to 740 quarters of corn and 25 tons of steel. The surface area of uniform land available is also fixed and equal to 6 acres.

Steel is produced by just one method, employing 2 units of labour per ton of output. Corn can instead be obtained by two methods,

α and β. With method α, 100 quarters of corn are produced per acre of land by the employment of 10 units of labour, whereas 185 quarters of corn per acre of land are obtained by employing 20 units of labour with method β.[6]

If the rate of rent were zero, method α would be cheaper than β because less labour is employed per quarter of corn. Due to the scarcity of land, however, the use of this method alone would allow the production of only 600 quarters of corn as against the 740 required by the market. Methods α and β must therefore necessarily coexist and the rate of rent must rise in order to make the unit costs identical within the two methods,[7] a task that it could not accomplish if rent did not enter into the costs.

When the rent reaches this level, the 6 acres of land will be entirely cultivated: 4.35 acres (4 acres, 1 rood and 16 perches) with method α and 1.65 acres (1 acre, 2 roods and 24 perches) with method β.

In this case, given a wage rate w, the profit rate r, the prices p_c and p_s and the rent rate ρ must be such that:[8]

$$20\,w\,(1 + r) + \rho = 185\,p_c \tag{1}$$

$$10\,w\,(1 + r) + \rho = 100\,p_c \tag{2}$$

$$2\,w\,(1 + r) = p_s \tag{3}$$

By solving system (1)-(3), we obtain the result that the rent rate ρ is equal to the value of 15 quarters of corn, as in Ricardo's example, and the relative price of corn in terms of steel is 1/17.

We must now determine the amount of labour embodied in corn relative to the amount embodied in steel so as to compare this value with the relative price. If Ricardo's argument about intensive rent is right, they must be equal. If instead they differ, this is due to the fact that rent enters into the price.

The usual determination of the labour time embodied in the commodities does not apply in the case considered here, which involves the more general problem of calculating the amounts of labour embodied in cases where more than one method of production is in use in the same industry. This issue has already received some attention (e.g., Morishima, 1973, Flaschel, 1983 and Toker, 1984).

According to a universally accepted basic definition, the amount of labour embodied in a commodity is the labour time employed directly in its production plus the labour embodied in all the commodities that are its means of production. It follows that if some commodities are produced by the simultaneous use of different

methods, there will be different amounts of labour embodied in each commodity. This is perfectly consistent, as we shall see, both with Marx's theory and with Ricardo's.

Let us start with Marx, who asserts that a commodity can have various "individual values" corresponding to the different methods of production in use. The differences between the individual values of the same commodities can arise from various circumstances, including the cultivation of different kinds of land (cf. Marx, 1909, p. 948), differences in the technical knowledge and, more simply, the fat that two or more methods are equally profitable, as at a switch point between two techniques or in the case of intensive rent addressed here. Each commodity also has a "market value" (or "social value"), however (cf. Marx, 1909, p. 210). In cases where the price of a commodity is regulated by the amount of labour it embodies, competition entails "the establishment of an equal market-value and market-price by averaging the various individual values of the commodities" (Marx, 1909, p. 212).

In Ricardo's analysis the distinction between individual and market value is less explicit but still present, at least when differential rent is considered. In fact, by saying that the "corn which is produced by the greatest quantity of labour is the regulator of the price of corn" (Ricardo, 1951-73: vol. 1, p. 46), Ricardo is implicitly arguing that while corn can embody different amounts of labour, it has just one price, which is equal to the greatest of these.[9] Therefore, in the case under consideration, Ricardo's view differs from Marx's precisely because the market value is the highest individual value in the former and the average of the individual values in the latter.[10]

Let us now return to our example. Due to the absence of capital goods, the calculation is very simple. The individual amounts of labour embodied in a quarter of corn with methods α and β are respectively $1/10$ and $4/37$ (= $20/185$), and the labour embodied in a ton of steel is 2. The market or social labour value of corn relative to that of steel is $2/37$ and therefore smaller than the relative price ($2/34$).[11] Rent does enter into the price of corn and causes an increase with respect to the level corresponding to the relative quantities of labour embodied.

ADDITIONAL REMARKS

Two further point can be noted before going on to state our conclusions. The first concerns the possibility of calculating the amount of labour embodied in commodities by means of an indirect method[12] and in particular of following the procedure Sraffa suggested with reference

to the case of joint production. In the example we are considering, this consists in comparing the amounts of labour used to produce 740 and 739 quarters of corn. According to Sraffa's argument, the difference between them will be equal to the labour embodied in the last quarter of corn (cf. Sraffa, 1960, p. 57).[13]

As can be easily verified, the production of 740 quarters of corn on 6 acres of land involves the employment of 76.471 units of labour in the corn sector[14] and the production of 739 quarters on the same surface of land instead involves the employment of 76.353 units.[15] The resulting difference between the quantities of labour employed is thus 0.118 or 4/34. Given that 2 units of labour are embodied in a ton of steel, it therefore follows that the ratio of the quantities of labour embodied, which is 2/34, is equal to the relative price. This result calls, however, for closer examination.

First, the increase in the amount of labour employed in order to produce one additional quarter of corn (i.e., 4/34) is greater than the amount needed to obtain one quarter of corn both with method α (1/10) and with method β (4/37). The explanation of this fact is quite simple. In order to obtain one more unit of corn with method β, we need 1/185 acres of land, but since all the land is already being used, this area is made available by reducing the use of the more land-intensive method, namely α.[16]

Second, it seems difficult to regard the difference between the increase in the amount of labour employed for the last unit of corn (4/34) and the amount directly embodied in that unit (4/37) as embodied labour. In actual fact, it is not labour either directly or indirectly embodied in corn, since capital consists exclusively in our example of wages paid in advance. How can this difference (4/34 − 4/37) enter into the cost of corn?

The solution seems to be quite simple. As noted in our first observation, 4/34 is not only the labour embodied in the last quarter of corn but also includes the additional labour embodied in the other 739 quarters. This additional labour enters into the production cost of the last quarter of corn – as well as every other quarter – in the following way. The production of one unit of corn with method β involves 4/37 (= 20/185) units of direct labour and 1/185 acres of land, and since the rent for an acre of land is the value of 15 quarters of corn, then the rent to be paid on 1/185 acres of land is the value of 3/37 (= 15/185) quarters of corn. If these 3/37 quarters of corn were considered not as rent, which they are, but as seed, then the labour directly and indirectly employed in one quarter of corn would be exactly 4/34.[17] To conclude,

the difference 4/34 – 4/37 should not be included in the amount of labour embodied in corn because it is the amount of labour embodied in the corn that pays the rent: an income and not an input.

The second remark concerns the possibility of generalising the result obtained in the third section by means of our example, in which capital consists exclusively of wages paid in advance at the beginning of the year. In that case, we proved that even under this favourable assumption, commodities are not exchanged at a ratio determined by their embodied quantities of labour when intensive differential rent is paid for the use of land, and that this therefore enters into the price of corn.

We can obtain the same conclusion by considering a case with a non-basic agricultural commodity and intensive rent, as found for example both in Montani (1975) and in Kurz (1978), and show that the latter enters into the price of the former.

In particular, let us follow Montani[18] and assume that there are two commodities: an industrial and basic commodity "a", which is also the numéraire, and an agricultural and non-basic commodity "z".

As regards technology, we have one method for the production of "a" and two different methods for the production of "z", namely α and β. The customary symbols, a_a and ℓ_a denote respectively the quantities of commodity "a" and of labour to be used in order to obtain one unit of "a", while a_z^t, ℓ_z^t and λ^t are the input coefficients of commodity "a", labour and land for a unit of "z" produced by method t, with $t = \alpha, \beta$ and $\lambda^\alpha > \lambda^\beta$ (i.e., method β gives a greater output per unit of land).

If both methods must be in use in order to meet the demand for "z" fully, then, given a post-factum wage rate w (or a profit rate r), the profit rate r (or the wage rate w), the rent rate ρ and the price of "z" p_z in terms of "a" are determined, according to Sraffa's theory, by solving the system:

$$a_a(1 + r) + \ell_a w = 1 \tag{6}$$

$$a_z^\alpha(1 + r) + \ell_z^\alpha w + \lambda^\alpha \rho = p_z \tag{7}$$

$$a_z^\beta(1 + r) + \ell_z^\beta w + \lambda^\beta \rho = p_z \tag{8}$$

Given the wage rate level w^*, and having determined the corresponding profit rate r^* by means of equation (6), we can therefore use equations (7) and (8) – as both Montani and Kurz do – in order to obtain the "price-rent relation" implied by each of the two methods. Skipping the details, for which readers are referred to the cited articles of Montani and Kurz, and denoting by c^α and c^β the unit cost of
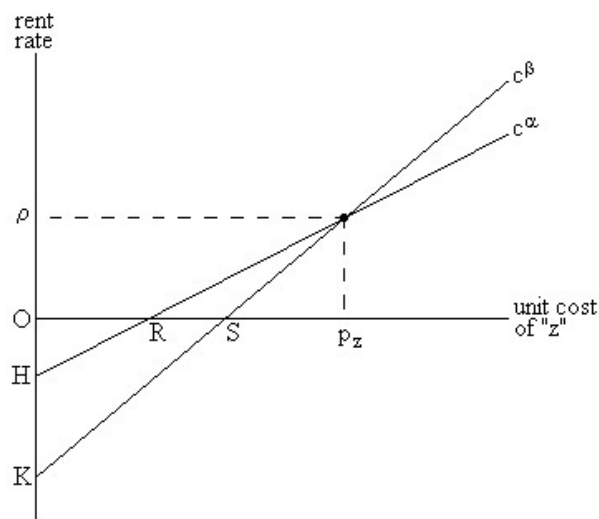
Figure 1

production[19] of "z" using methods α and β respectively, we have the functions plotted in Fig. 1.

It is clear from Fig. 1 that intensive differential rent enters into the unit cost with both methods,[20] albeit in different amounts, and therefore into the price of "z" in terms of "a". The unit costs c^α and c^β are both increasing functions of the rate of rent, but because $\lambda^\alpha > \lambda^\beta$, c^α grows faster than c^β when this rate increases. There is therefore a positive rate of rent ρ that allows c^α to catch up with c^β and thus allows the two production methods to be simultaneously in use. Moreover, this rate makes the price p_z greater than the costs for wages and gross profits both with method α (i.e., the length of segment OR in Fig. 1) and with method β (the length of segment OS).

Therefore, contrary to Ricardo's claim that "no reduction would take place in the price of corn, although landlords should forego the whole of their rent" (Ricardo, 1951-1973: vol. I, pp. 74, 75), if $\rho = 0$ is posited, then both c^α and c^β will be reduced. In particular, in the case of zero rents, a price equal to OS will allow the producers to pay the workers, recoup the capital and make at least ordinary profits with both the methods.

CONCLUSION

Contrary to what Ricardo claims and many economists have since believed, if the last piece of land under cultivation receives intensive

differential rent, it enters into the price of corn or agricultural products in general.

This has been proved by means of Ricardo's own example. In particular, even assuming that capital in each sector consists exclusively of wages anticipated for one year, we have shown that in the presence of intensive differential rent, the relative price of commodities cannot be equal to the ratio of the quantity of labour they embody.

Malthus and Say were therefore right to argue that Ricardo's idea of rent not entering into the price of commodities is based on the presence of cultivated lands which pay no rent, a situation to be found only in very particular cases, such as a newly colonised country. But where all cultivated land pays a rent, which seems to be the general case, rent is an element of the cost and the price of agricultural commodities.

This result also rehabilitates an assertion made by Adam Smith and criticised by Ricardo. In countries where all the land is owned and all cultivated land pays a rent, the "original rule" is in fact altered in that the price of commodities is no longer determined by the relative quantity of labour embodied.

Acknowledgments

Notes

1. Samuelson (1959, p. 9, footnote 2) proposed a reformulation of the Ricardian theory of rent in terms of differential calculus and marginal productivity. In this case, assuming a continuum of different qualities of land – as in von Thunen theory for example – the extensive differential rent would be compatible with a situation in which almost every cultivated land yields a rent to its proprietor. But clearly this was not the case considered by Ricardo, since he explicitly referred to the intensive rent.

2. Cf. in particular, Ricardo (1951-73) vol. 1, p. 413; vol. 7, p. 372 and vol. 8, pp. 149, 150.

3. Authoritative examples include Marshall (1893, p. 86) and S. Hollander (1979, p. 203). Nevertheless, Hollander (1991, p. 162) presented a peculiar view, claiming that Ricardo was 'a "demand-supply" value theorist' and that in his theory the 'price ratio is proportionate to marginal labour cost' (*op. cit.*, p. 163). While discussion of these claims lies beyond the scope of the present paper, some further observations on marginal labour cost pricing applied to the case we are addressing will be made in the fourth section.

4. We refer in particular to the analysis described by Sraffa (1960: 75, 76 - §§ 87, 88) and put into formal terms and developed in greater depth by various

economists, including Montani (1975), Kurz (1978), Guichard (1982) and D'Agata (1983). Cf. also: Kurz and Salvadori (1995, p. 289).

5. For example, if capital became more productive with method α, so that £1,000 capital produced 110 quarters of corn per acre, the quantity of corn obtained with the second dose of capital would be reduced to 75 quarters because the quantity produced by method β (185 quarters per acre), being a well-defined method of production, is independent of any improvement in method α. As a result, the second dose of capital cannot be regarded as employed with a third independent method.

6. It should be noted that if w = £100, method α makes it possible to produce 100 quarters per acre with a capital of £1000 and method β to produce 85 quarters more per acre through a further investment of £1000, as in Ricardo's own example.

7. Until 740 quarters of corn are produced, the price of corn is above its cost of production with method α and agricultural producers therefore obtain extra profits. Since the six acres of land are entirely cultivated, the classical mechanism of capitalistic competition allows landowners to appropriate the extra profits in the form of rents. The rising rents tend to increase the unit cost of production with method α with respect to the unit cost with method β. This process goes on until the unit cost of production for corn is the same with both methods.

8. As regards the equations determining intensive differential rent simultaneously to prices, we consider the analysis put forward by Sraffa (1960, pp. 75, 76 - §§ 87, 88) and later put into formal terms by various authors (in particular, Montani, 1975, Kurz, 1978 and Kurz and Salvadori, 1995). Equations (1)-(3) are the same as though found there but written for the simple example addressed. It is also worth observing that although Sraffa and the scholars mentioned above have a correct theory of intensive differential rent, they fail to note that it can be used to refute Ricardo's conclusions about rent and the labour theory of value (for a discussion of the differences between Sraffa's and Ricardo's theory of rent see also Fratini, 2008).

9. On the contrary, according to Marx: "[i] t is only extraordinary combinations of circumstances under which commodities produced under the least or most favorable conditions regulate the market-value." Marx 1909, p. 210.

10. According to the analyses of Flashel (1983) and Toker (1984), this point can be stated in formal terms as follows. Let us assume there are n commodities – labelled 1, 2, ..., n – and each one of them can be produced by many methods. Let us denote by b_{jk} the amount of commodity j (with j = 1, 2, ..., n) produced with method k, and by a_{ijk} and ℓ_{jk} respectively the amounts of commodity i and labour employed in that activity. If s_j is the number of activities simultaneously in use for the production of commodity j, then the individual values v_{jk} (with k = 1, 2, ..., s_j) and the social or market value v_j must satisfy the following equations:

$$v_{jk} \cdot b_{jk} = \sum_{i=1}^{n} v_i \cdot a_{ijk} + \ell_{jk} \qquad \forall j = 1, 2, ..., n, \ \forall k = 1, 2, ..., s_j.$$

The above system has $\sum_j s_j$ equations and $\sum_j s_j + n$ unknowns. Therefore, in order to close the system, we need n more equations. In the case of Marx's conception of market value, these are (cf. Flashel, 1983, p. 442, and Toker, 1984, p. 152):

$$v_j \sum_{k=1}^{s_j} b_{jk} = \sum_{k=1}^{s_j} v_{jk} \cdot b_{jk} \qquad \forall j = 1, 2, ..., n.$$

In the case of Ricardo, they can instead be written as follows:

$$v_j = \max \{v_{j1}, v_{j2}, ..., v_{js_j}\} \qquad \forall j = 1, 2, ..., n.$$

11. It is worth observing that, as Sraffa points out (1960, p. 75), the relative price of corn in terms of steel can be determined by a system in which land does not appear. This possibility has been studied in general and formal terms by Guichard (1982). Since this way of framing the equations might generate a sort of optical illusion, some clarification will be necessary in order to avoid possible misunderstanding. As regards the example addressed here, equations (1)-(3) are written according to the fundamental principle lying at the basis of Sraffa's price equations, as well as similar equations found in Walras's theory (see Fratini & Levrero, 2011), i.e., that every sector of the economy must earn zero extra-profits and commodity prices must therefore be equal to their average costs of production (including the ordinary profit on the capital per unit of product). Once this economic principle is used to write the equations, however, the system can then be addressed with every tool made available by mathematics. In particular, it is undoubtedly mathematically sound to use equations (1) and (2) in order to obtain:

$$10 \, w \, (1 + r) = 85 \, p_c. \tag{4}$$

In doing this, we have used an equation in order to eliminate a variable, namely the rate of rent, as is usual when solving a system "by substitution". Moreover, equations (3) and (4) form a system whose solution – for a given wage rate – makes it possible to obtain the relative price of the two commodities and the rate of profit.

This way of solving the equations may, however, generate the optical illusion mentioned above. The fact that the rate of rent does not appear in equations (3) and (4) might seem to suggest that the relative price is not affected by it. This is not true, as the relative price depends on the relative average cost of the two commodities, and the average cost of corn *is* affected by the rate of rent. More precisely, if the rate of rent did not make the average cost of corn production with method α equal to that with method β, equation (4) could not exist, as is clear from the way in which it is obtained. The price determined by equation (4) therefore includes the rate of rent even if it does not appear.

Moreover, great care must be taken not to infer from equation (4) that 10 units of labour are embodied in 85 quarters of corn. This would be true if 85 quarters were the gross product obtained by the employment of 10 workers with the least favourable method, i.e., if equation (4) had exactly the same

economic meaning as equation (1). But this is not so. The 85 quarters of corn in equation (4) are not the gross product of 10 workers (which is instead 100 quarters with method α and 92.5 with method β) but a physical amount of gross profits. Equation (4) tells us only that a capital of $10\,w$ employed in agriculture must earn the value of 85 quarters of corn as gross profits regardless of the method used. This fact is also proved by observing that while equations (1) and (2) can similarly be used in order to obtain

$$\rho = 15\,p_c \tag{5}$$

we cannot infer from this either that wages and profits do not affect the price of corn or that one quarter of corn "embodies" 1/15 acres of land. Equation (5) is instead what allows Ricardo to claim that the rent for an acre is the value of 15 quarters of corn.

12. As is known, the most common indirect method to calculate the relative labour embodied in commodities in the usual cases – where the number of methods in use is equal to that of commodities and there is no joint production – consists in determining relative prices in cases where wages are the only form of income.

    If we apply this procedure to our equations (1)–(3) and posit that $r = \rho = 0$, it is clear that the system is over-determined. Once steel is adopted as the numéraire commodity, the system has three independent equations and only two unknowns, namely the wage rate w and the price of corn in terms of steel. This problem can be avoided, however, by the expedient of allowing the prices (in terms of steel) of the corn produced with methods α and β — labelled respectively p_c^α and p_c^β to differ. When this is done, the solution of the system is $w = 1/2$, $p_c^\alpha = 1/20$ and $p_c^\beta = 2/37$; the two prices of corn are thus nothing other than the relative individual values of corn in terms of steel that we have already determined in the third section.

13. The use of this procedure to calculate the labour embodied by the commodities in the case of joint production was severely criticised by Steedman (1976 and 1977) and Toker (1984) because, as Sraffa himself admitted, it can imply negative quantities of labour embodied for some commodity.

14. If C is the quantity of corn to be produced, C_α and C_β are the quantities produced respectively by means of methods α and β, and 6 acres is the available surface of land, this necessarily gives us:

    $$C_\alpha + C_\beta = C$$

    $$C_\alpha/100 + C_\beta/185 = 6.$$

    435.29 quarters are thus to be obtained by means of method α and 304.71 by means of method β. It follows that the amount of labour employed in the corn sector is $435.29(10/100) + 304.71(20/185) = 76.471$.

15. Following the same argument as in the previous note, if 739 quarters are to be produced, then the quantities of corn obtained with methods α and β are respectively 436.47 and 302.53, and the labour employed is $436.47(10/100) + 302.53(20/185) = 76.353$.

16. The quantity of corn obtained with method α drops from 436.47 quarters to 435.29, while that obtained with method β increases by more than one unit from 302.53 to 304.71.

17. As is well known, if a quarter of corn is produced with 4/37 units of direct labour and 3/37 quarters of corn, then the amount of labour embodied in it is the solution of the following equation:

$$4/37 + x \, 3/37 = x$$

and therefore $x = 4/34$.

18. A similar case can be found in Kurz (1978), pp. 26-28.

19. It is worth noting that Montani (1975, p. 81) – as well as Kurz (1978, p. 27) – does not include the intensive differential rent in the unit cost of production of commodity "z". He is therefore considering a concept of cost that excludes entrepreneur's revenues and assuming (implicitly) that landowners organise production, since rents are viewed as a residuum in this case. When the concept of full cost is adopted, however, as it is here, rent must be included in the cost of production of commodity "z" (even when landowners are entrepreneurs).

20. While our Fig. 1 is identical to Fig. 4 in Montani (1975, p. 81) and Fig. 3 in Kurz (1978, p. 28), they seem to use their figures solely in order to show the existence, under certain conditions, of economically meaningful solutions for equations (6), (7) and (8). The same figure is used here for a different purpose.

References

D'Agata A., (1983), The Existence and Uniqueness of Cost-Minimizing Systems in Intensive Rent Theory, *Metroeconomica*, Vol. 35, Pp. 147-158.

Flaschel P., (1983), Actual Labour Values in a General Model of Production, *Econometrica*, Vol. 51, Pp. 435-454.

Fratini S. M., (2008), Alcune Osservazioni Sulla Teoria Classica Della Rendita, *QA – Rivista dell'Associazione Rossi-Doria*, Pp. 131-158.

Fratini S. M., and Levrero E. S., (2011), Sraffian Indeterminacy: A Critical Discussion, *Cambridge Journal of Economics*, Vol. 35, Pp. 1127-1149.

Guichard J. P., (1982), La Rente Differentielle Intensive, Expression d'un Processus d'Intensification des Cultures, In: R. Arena *et al.*, (Eds), *Études d'economie Classique et Neoricardienne*, Paris: Presses Universitaires de France.

Hollander S., (1979), *The Economics of David Ricardo*, London: Heinemann Educational Books.

Hollander S., (1991), On the Endogeneity of the Margin and Related Issues in Ricardian Economics, *Journal of the History of Economic Thought*, Vol. 13, Pp. 159-174.

Kurz H., (1978), Rent Theory in a Multisectoral Model, *Oxford Economic Papers*, Vol. 30, Pp. 16-37.

Kurz H., and Salvadori N., (1995), *Theory of Production*, *A Long-Period Analysis*, Cambridge: Cambridge University Press.

Marshall A., (1893), On Rent, *The Economic Journal*, Vol. 3, Pp. 74-90.

Marx K., (1909), *Capital Vol. III*, Chicago: Charles H. Kerr & Co.

Montani G., (1975), Scarce Natural Resources and Income Distribution, *Metroeconomica*, Vol. 27, Pp. 68-101.

Morishima M., (1973), Marx in the Light of Modern Economic Theory, *Econometrica*, Vol. 42, Pp. 611-632.

Ricardo D., (1951-1973), *Works and Correspondence of David Ricardo*, P. Sraffa (Ed.), Vol. 11, Cambridge: Cambridge University Press.

Samuelson P. A., (1959), A Modern Treatment of the Ricardian Economy: I. The Pricing of Goods and of Labor and Land Services, *The Quarterly Journal of Economics*, Vol. 73, Pp. 1-35.

Smith A., (1976), *The Glasgow Edition of the Works and Correspondence of Adam Smith*, Vol. 7, Oxford: Clarendon Press.

Sraffa P., (1960), *Production of Commodities by Means of Commodities*, Cambridge: Cambridge University Press.

Steedman I., (1976), Positive Profits with Negative Surplus Value, *The Economic Journal*, Vol. 85, Pp. 114-123.

Steedman I., (1977), *Marx After Sraffa*, London: New Left Books.

Toker M. A., (1984), A Note on 'Negative' Quantities of Embodied Labour, *The Economic Journal*, Vol. 94, Pp. 149-154.