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# A note on re-switching and the neo-Austrian concept of the average period of production

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## I.1 Introduction

The neo-Austrian average period of production: introduced by Hicks in *Value and Capital* (1939) and further developed in Sargan (1955), von Weizsäcker (1971 and 1977) and Malinvaud (1986 and 2003).

This new concept of the average period is grounded on the idea of using the **shares of costs** anticipated at each date out of the total cost of production as weights instead of the **shares of labour** performed in each period out of the total labour embodied.

Once this notion has been adopted, if a fall in the rate of interest brings about a change in the method of production of a certain commodity, then the incoming method is more 'roundabout' than the outgoing.

## I.2 Introduction

Cachanosky and Lewin (2014) claim that the adoption of this new average period of production 'adds plausibility' to the **Austrian business-cycle theory**, which they describe as follows:

> a monetary policy that reduces interest rates, increases the 'average period of production', or the degree of 'roundaboutness,' of the 'structure of production,' that is out of sync with consumer preferences, thus creating unsustainable imbalances in that structure. The increase in 'roundaboutness,' followed by its reduction when the monetary authority revises interest rates upward, is what constitutes the boom and bust in this business-cycle theory. (Cachanosky and Lewin, 2014, p. 648.)

This paper seeks to ascertain whether the new neo-Austrian concept of the average period of production can actually prove useful in arguments of this kind.

## II.1 The old concept of the average period of production

A final commodity such as corn can be obtained by two different and alternative methods of production: **method A** and **method B**.

Method A

| **Periods** | $t$ - 2 | $t$ - 1 | $t$ |
|---|---|---|---|
| **Inputs (labour)** | $a_3$ | $a_2$ | $a_1$ |
| **Outputs (corn)** | 0 | 0 | 1 |

Method B

| **Periods** | $t$ - 2 | $t$ - 1 | $t$ |
|---|---|---|---|
| **Inputs (labour)** | $b_3$ | $b_2$ | $b_1$ |
| **Outputs (corn)** | 0 | 0 | 1 |

## II.2 The old concept of the average period of production

**Labour embodied in one unit of corn**

Method A: $a_1 + a_2 + a_3$. Method B: $b_1 + b_2 + b_3$.

**The average period of production (traditional conception)**

$$T_a = \frac{a_1+2a_2+3a_3}{a_1+a_2+a_3} \qquad (1)$$

$$T_b = \frac{b_1+2b_2+3b_3}{b_1+b_2+b_3} \qquad (2)$$

[It is assumed that $a_1 + a_2 + a_3 < b_1 + b_2 + b_3$ and $T_a > T_b$]

## II.3 The old concept of the average period of production

**A.1**. wages are paid at the beginning of each period; **A.2**. value is measured in terms of labour commanded and, hence, the wage rate is set equal to 1; **A.3**. interest is determined by means of the simple interest formula at the rate $r$.

Let us denote by $p_a$ and $p_b$ the unit costs of production (in terms of labour commanded) of corn with the two methods. Under the assumptions A.1−A.3, we have:

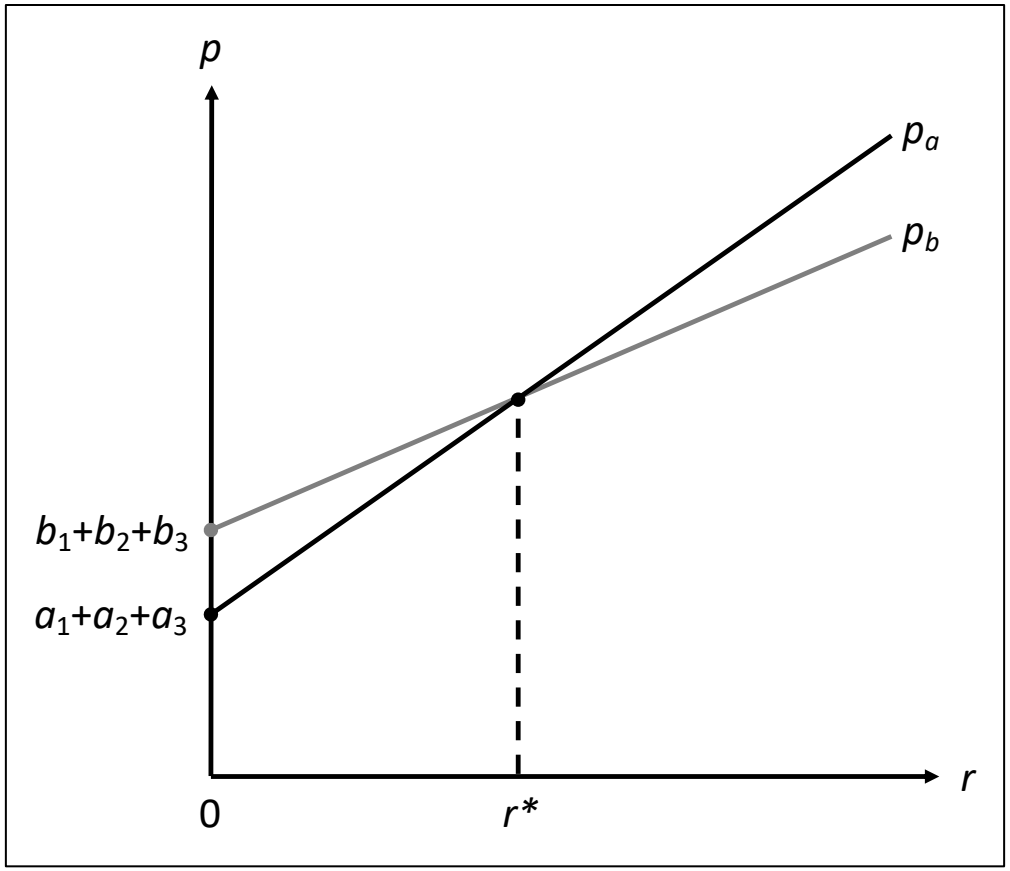
$$p_a = a_1 (1 + r) + a_2 (1 + 2\,r) + a_3 (1 + 3\,r) = (a_1 + a_2 + a_3) (1 + T_a\,r) \quad (3)$$

$$p_b = b_1 (1 + r) + b_2 (1 + 2\,r) + b_3 (1 + 3\,r) = (b_1 + b_2 + b_3) (1 + T_b\,r) \quad (4)$$

**Proposition 1**. For a given method, the amount of interest paid per unit of labour is proportional to the average period of production.

## II.4 The old concept of the average period of production

**Proposition 2**. When a rise in the rate of interest entails a change of the method in use, the incoming method has an average period of production shorter than the outgoing one.



## III.1 The new concept of the average period of production

If the compound interest formula is adopted, then the unit costs must be redefined as follows:

$$p_a = a_1(1+r) + a_2(1+r)^2 + a_3(1+r)^3 = (1+r)[a_1 + a_2(1+r) + a_3(1+r)^2] \quad (5)$$

$$p_b = b_1(1+r) + b_2(1+r)^2 + b_3(1+r)^3 = (1+r)[b_1 + b_2(1+r) + b_3(1+r)^2] \quad (6)$$

### The average period of production (new conception)

$$\Theta_a = \frac{a_1(1+r)+2a_2(1+r)^2+3a_3(1+r)^3}{a_1(1+r)+a_2(1+r)^2+a_3(1+r)^3} = \frac{a_1+2a_2(1+r)+3a_3(1+r)^2}{a_1+a_2(1+r)+a_3(1+r)^2} \quad (7)$$

$$\Theta_b = \frac{b_1(1+r)+2b_2(1+r)^2+3b_3(1+r)^3}{b_1(1+r)+b_2(1+r)^2+b_3(1+r)^3} = \frac{b_1+2b_2(1+r)+3b_3(1+r)^2}{b_1+b_2(1+r)+b_3(1+r)^2} \quad (8)$$

## III.2 The new concept of the average period of production

Once this new definition has been adopted, the connection between the average period of production and the rate of interest is twofold:

- on the one hand, $\Theta_a$ and $\Theta_b$ depend on the rate of interest;
- on the other, they express the **elasticity** of the unit costs $p_a$ and $p_b$ with respect to the interest factor $R \equiv (1 + r)$.

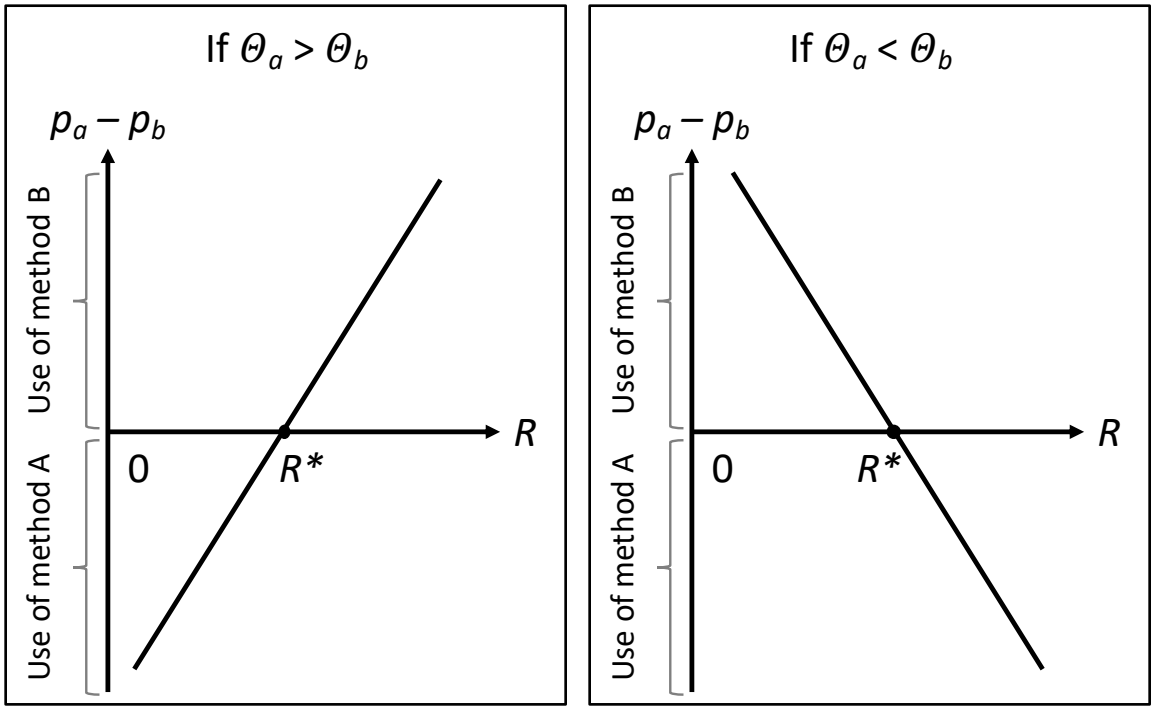
$$\frac{dp_a/dR}{p_a/R} = \frac{a_1+2a_2R+3a_3R^2}{a_1+a_2R+a_3R^2} = \Theta_a \qquad (9)$$

$$\frac{dp_b/dR}{p_b/R} = \frac{b_1+2b_2R+3b_3R^2}{b_1+b_2R+b_3R^2} = \Theta_b. \qquad (10)$$

The relative cost of the method with the longest average period of production therefore rises as the rate of interest increases.

## III.3 The new concept of the average period of production

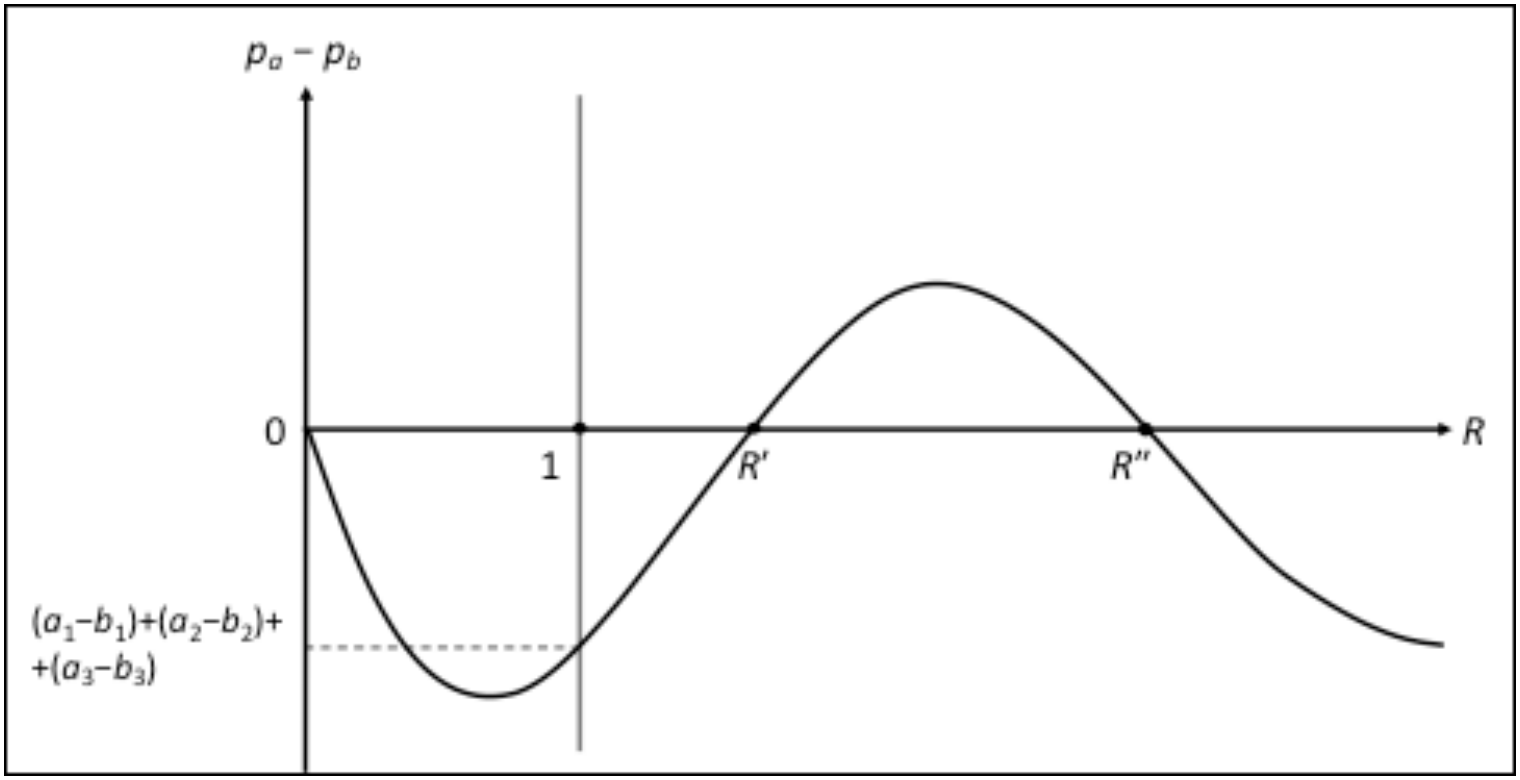
Let $R^*$ be an interest factor such that $p_a(R^*) - p_b(R^*) = 0$. In a neighbourhood of $R^*$, $\Theta_a > \Theta_b$ implies that the difference $p_a - p_b$ rises as the interest factor increases and the rise of the interest factor leads to the abandonment of method A and the use of method B. Similarly, $\Theta_a < \Theta_b$ means that the difference $p_a - p_b$ falls as the interest factor increases and method B is therefore abandoned in favour of method A.



## IV.1 Re-switching

Let us calculate the difference in unit costs from equations (5) and (6):

$$p_a - p_b = R\,[(a_1 - b_1) + (a_2 - b_2)\,R + (a_3 - b_3)\,R^2] \qquad (11)$$



## IV.2 Re-switching

Methods with longer average periods can give a smaller final output per worker.

If $y_a$ and $y_b$ are the corn obtained per worker employed respectively with method A and B, then:

$$y_a = \frac{1}{a_1+a_2+a_3} \tag{12}$$

$$y_b = \frac{1}{b_1+b_2+b_3} \tag{13}$$

Therefore, $a_1 + a_2 + a_3 < b_1 + b_2 + b_3$ implies $y_a > y_b$.

**In a neighbourhood of *R*", even though an increase in the rate of interest brings the method with the shortest period into use (method A), it entails not a fall but a rise in the final output per worker.**

## V.1 Conclusion

The Austrian business-cycle theory is grounded on the following relationships:

(**a**) an inverse relationship between the rate of interest and the degree of roundaboutness of the production processes;

(**b**) a direct relationship, ceteris paribus, between the degree of roundaboutness of the production processes and the level of output.

If (a) and (b) hold, then a fall in the rate of interest due to the adoption of an expansive monetary policy prompts an artificial boom and the return toward the equilibrium position leads to a bust.

## V.1 Conclusion

On the one hand, relationship (a) always holds when the new concept of the average period is adopted (with compound interest capitalisation).

On the other, however, relationship (b) fails in the event of re-switching, as the most roundabout method of production gives the smallest output per worker in a neighbourhood of the second switching point.

In this case, a fall in the rate of interest would bring about an increase in the degree of roundaboutness but not a boom. Contrary to what the Austrian business-cycle theory predicts, it would entail a bust.

In the light of this result, the new notion of the average period of production does not appear to provide adequate support for the Austrian business-cycle theory. In particular, this business-cycle theory would actually appear to require the faulty conception of capital that these neo-Austrian scholars are rightly endeavouring to avoid.

# Thank you!

There are certain unsettled questions in economic theory that have been handed down as a sort of legacy from one generation to another. … Not unfrequently the discussion is carried far beyond the limits of weariness and satiety, so that it may well be regarded as an offence against good taste to again recur to so well-worn a theme. And yet these questions return again and again, like troubled spirits doomed restlessly to wander until the hour of their deliverance shall appear.
(Böhm-Bawerk 1884, p. 149)