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NEOCLASSICAL THEORIES OF STATIONARY RELATIVE PRICES AND THE SUPPLY OF CAPITAL

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I.1 The rate of interest in neoclassical theories

In early neoclassical theories, the distributive variables: wage rate, rent rate and rate of interest, were understood as the prices firms have to pay for the employment of the factors of production: labour, land and capital.

Classical approach		
3 social cl. → 3 kinds of income		
workers	→	wages
capitalists	→	profits
landlords	→	rents

Neoclassical approach		
3 kinds of income → 3 factors of prod.		
wages	→	labour
interest	→	capital
rents	→	land

I.2 The rate of interest in neoclassical theories

Two agents: **Houseolds** and **Firms**

Households demand the outputs (commodities) and supply the factors of production (labour, capital and land). Firms demand the factors of production and supply the outputs.

Incomes from Capital

I.3 The rate of interest in neoclassical theories

The rate of interest was intended as a variable reacting to discrepancies between supply of and demand for capital, in more or less the same way as the price of a commodity reacts to the difference between its supply and demand.

The equilibrium level of the rate of interest was thought to involve the equality between the quantity of capital demanded by firms and the stock of capital supplied by households:

> interest, being the price paid for the use of capital in any market, tends towards an equilibrium level such that the aggregate demand for capital in that market, at that rate of interest, is equal to the aggregate stock forthcoming there at that rate (Marshall 1920, p. 534).

I.4 The rate of interest in neoclassical theories

Since the available quantities of labour and land were considered exogenous magnitudes within the theory, similarly, the **existing stock of capital**—understood as either an amount of value or an endowment of capital goods—was taken as a given amount.

According to Marshall (1920, p. 534), for instance, “it is only slowly and gradually that the rise in the rate of interest will increase the total stock of capital”, so that, for the purposes of the theory of value and distribution, capital accumulation could be neglected.

II.1 End of the idea of capital as a factor of production

Within the **neo-Walrasian** general equilibrium theory, capital is not understood as a factor of production and the rate of interest in not regarded as the price firms have to pay for its use.

This is admitted by **Samuelson** (1966, p. 582):

> There often turns out to be no unambiguous way of characterizing different processes as more "capital-intensive," more "mechanized," more "roundabout," except in the *ex post* tautological sense of being adopted at a lower interest rate and involving a higher real wage. Such a tautological labeling is shown, in the case of reswitching, to lead to inconsistent ranking between pairs of unchanged technologies, depending upon which interest rate happens to prevail in the market. (p. 582)

In general, we cannot say that a technique or a method of production is more "capital-intensive" than another. The ranking of techniques on the basis of their "capital-intensity" depends upon the rate of interest.

II.2 End of the idea of capital as a factor of production

Similarly, **Hahn** claims (1975, p.):

> The neo-Ricardians, by means of the neoclassical theory of the choice of technique, have established that capital aggregation is theoretically unsound. Fine. Let us give them an alpha for this. The result has no bearing on the mainstream of neoclassical theory simply because it does not use aggregates. It has a bearing on the vulgar theories of textbooks. But textbooks are not the frontier of knowledge.

The idea of (aggregate) capital as a factor of production is theoretically unsound, but the modern neo-Walrasian theory ("the frontier of knowledge") provides a supply-and-demand approach to value and distribution that is completely independent from the idea of capital as a factor of production.

II.3 End of the idea of capital as a factor of production

Then, the given stock of capital that characterized the initial versions of the neoclassical theory has been interpreted as due to a ‘**missing equation**’ in those equilibrium systems.

In particular, re-reading those early attempts from a neo-Walrasian standpoint, various scholars identified the missing equation with a **condition of zero net savings**, which is required by the stationarity of the system and which relates intertemporal households’ decisions about current and future consumption with firms’ choices of the optimal production plans.

II.3 End of the idea of capital as a factor of production

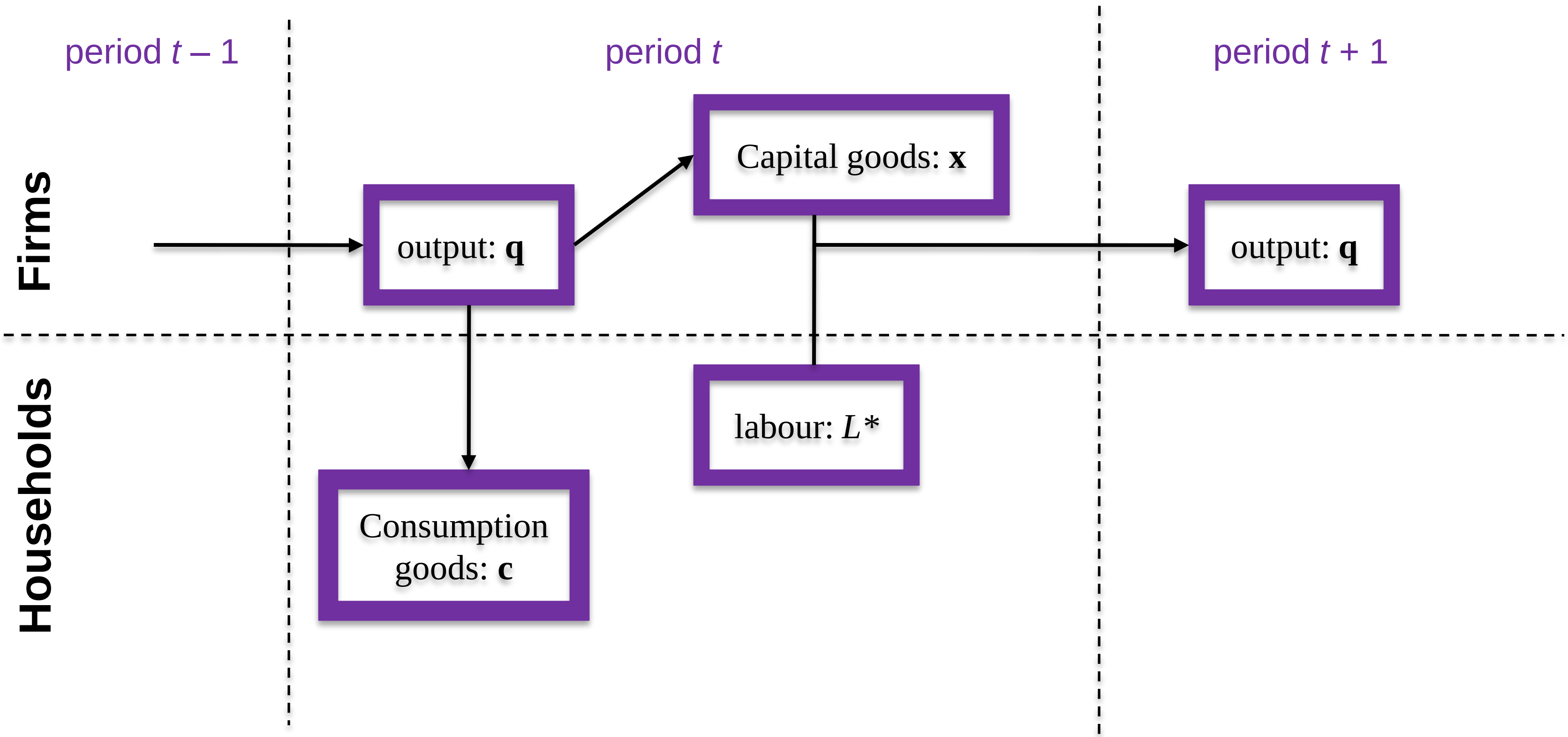
As Hicks wrote (1946, pp. 118-119):

> since we are in a stationary state, there can be no tendency for the stock of capital to increase or diminish; […] if entrepreneurs do not desire to increase or diminish their stock, their net borrowing must be nil. If the demand and supply for loans are to be in equilibrium, net saving must therefore also be nil. The rate of interest must therefore be fixed at a level which offers no incentive for net saving or dis-saving. What this level is depends partly upon the propensities to save of the individuals composing the community, partly upon their real incomes—and these depend again upon the size of the stock of intermediate products.

Here, we shall start from the neo-Walrasian stationary equilibrium in order to use it as a benchmark in the analysis of the early equilibrium models (Wicksellian model).

III.1 The neo-Walrasian stationary model

Semi-stationary model (Malinvaud 1953, section IV, and Bliss 1975, chapter 4): It is a recursive production model in which every period is identical to both the previous and the following period.



III.2 The neo-Walrasian stationary model

There are N different commodities

A vector of outputs $\mathbf{q} \in \Re^N$ emerges at each date from the processes started in the previous period.

Part of these outputs, namely a vector $\mathbf{c} \in \Re^N$, is consumed by households during the period.

The other part $\mathbf{x} = \mathbf{q} - \mathbf{c}$ is made up of the commodities employed as inputs together with the available labour force L^*.

The employment of $\mathbf{x}$ and L^* will give a vector of outputs $\mathbf{q}$ in the subsequent period.

IV.1 The consumption side

- L^* identical individuals are born at each date.

- They live for two periods: youth and old age.

- At birth, each individual has no other endowment than a unit of labour services to perform during youth.

Accordingly, consumption during the second period of life depends on saving decisions taken in the first period.

There are **overlapping generations**.

IV.2 The consumption side

Let $\mathbf{p} \in \Re^N$ be a stationary price vector and w and r be the wage and the interest rate respectively, an individual i, with $i = 1, 2, \ldots, L$, decides the consumption plan so as to maximise her or his intertemporal utility subject to the **budget constraint**:

$$\mathbf{c}_{i1}^{\mathrm{T}}\,\mathbf{p} + \mathbf{c}_{i2}^{\mathrm{T}}\,\mathbf{p}\,(1 + \mathrm{r})^{-1} \leq w \qquad (1)$$

where $\mathbf{c}_{ij} \in \Re^N$, with $j = 1, 2$, is the bundle of commodities consumed by the individual during her or his j-th period of life.

IV.3 The consumption side

Individual demand functions for consumption goods arise from the solution of the utility maximisation problem:

$\mathbf{c}_{i1}(\mathbf{p},w,r)$: demand for consumption goods delivered in the first period of life of individual i

$\mathbf{c}_{i2}(\mathbf{p},w,r)$: demand for consumption goods delivered in the second period of life of individual i

The demand for consumption goods in each period is the sum of the demand from the young generation born in that period and the one from the generation born in the previous period, who are now in old age:

$$\mathbf{c}(\mathbf{p},w,r) := \Sigma_i\, \mathbf{c}_{i1}(\mathbf{p},w,r) + \Sigma_i\, \mathbf{c}_{i2}(\mathbf{p},w,r) \qquad (2)$$

IV.4 The consumption side

There is no saving by elderly people.

In each period, the total amount of gross savings corresponds to the difference between income and consumption expenditure of the young generation:

$$s(\mathbf{p},w,r) := L^* \,\mathrm{w} - \left[\Sigma_i \, \mathbf{c}_{i1}(\mathbf{p},w,r)\right]^{\mathrm{T}} \mathbf{p} \qquad (3)$$

V.1 The production side

$\mathbf{q} \in \Re^N$: vector of commodity outputs

$\mathbf{x} \in \Re^N$ and L : inputs of commodities and labour services

$(\mathbf{q},\mathbf{x},L) \in \Re^{2N+1}$: production plan

We assume there is a differentiable transformation function $\varphi : \Re^{2N+1} \rightarrow \Re$ such that $Y := \{(\mathbf{q},\mathbf{x},L) \in \Re^{2N+1} : \varphi(\mathbf{q},\mathbf{x},L) = 0\}$ is the **set of technically feasible production plans**.

V.2 The production side

Profit maximization problem

$$\max \quad \mathbf{q}^{\mathrm{T}}\mathbf{p} - \mathbf{x}^{\mathrm{T}}\mathbf{p}\,(1 + \mathrm{r}) - L\,w,$$
$$\text{sub.to} \quad \varphi(\mathbf{q},\mathbf{x},L) = 0$$

First order conditions:

$$p_n - \lambda \frac{\partial \varphi(\mathbf{q},\mathbf{x},L)}{\partial q_n} = 0 \qquad n = 1, 2, \ldots, N$$

$$-p_n\,(1 + r) - \lambda \frac{\partial \varphi(\mathbf{q},\mathbf{x},L)}{\partial x_n} = 0 \qquad n = 1, 2, \ldots, N$$

$$-w - \lambda \frac{\partial \varphi(\mathbf{q},\mathbf{x},L)}{\partial L} = 0$$

$$\varphi(\mathbf{q},\mathbf{x},L) = 0$$

VI.1 Equilibrium conditions

Market-clearing conditions for commodities and labour services:

$$\mathbf{c}(\mathbf{p},w,r) = \mathbf{q} - \mathbf{x}$$

$$L = L^*$$

First order conditions

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$$\varphi(\mathbf{q},\mathbf{x},L) = 0$$

Zero net capital accumulation condition:

$$s(\mathbf{p},w,r) - \mathbf{x}^{\mathrm{T}}\mathbf{p} = 0$$

VI.1 Equilibrium conditions

We have a system of $3N + 4$ equations with $3N + 4$ unknown equilibrium variables $(\mathbf{q},\mathbf{x},L,\lambda,\mathbf{p},w,r)$.

On the one hand, there are $3N + 3$ independent equations, due to Walras's law.

On the other hand, once a commodity is adopted as numéraire, the (relative) prices to determine are $N - 1$.

Accordingly, there are $3N + 3$ equations with $3N + 3$ unknown variables to determine.

VII.1 Capital as a factor of production

The founders of the marginalist/neoclassical theory of value and distribution conceived their general equilibrium systems in a different way.

Two main differences.

First, these authors understood capital as a factor of production on the same footing as labour (and land), so that capital and labour are even substitutable at the margin in the production processes. There must be, therefore, a **demand for capital** from firms that is similar to and connected with their demand for labour.

Second, the zero net-accumulation condition, which characterizes the stationary models, is re-interpreted in terms of a constant **stock of capital available** K^*.

VII.2 Capital as a factor of production

The basic idea behind the given endowment of capital K^* is well known.

Each individual, and accordingly the economy, is endowed with a stock of existing capital goods, which is a legacy of the past.

Since capital accumulation is assumed to be a very slow and gradual process, the amount of capital in value terms can be approximately considered as a given magnitude and this makes the model stationary.

Nonetheless, since the vector $\mathbf{x}$ of capital goods employed in equilibrium does not correspond, in general, to the stock inherited from the past, the quantities of the capital goods have to change while their total value remains constant and equal to K^*, namely the value of the existing capital goods.

As this change is typically a long-run phenomenon, its result is called long-run equilibrium.

VII.3 Capital as a factor of production

In every period, individual endowments are made up of a certain quantity of labour L_i* and a certain amount of capital K_i*.

A flow of net income $w L_i^* + r K_i^*$ springs from these endowments and is used to finance the consumption expenditure of each period, while individual gross savings are K_i* by assumption.

The single-period budget constraint is then:

$$\mathbf{c}_i^{\mathrm{T}} \mathbf{p} \leq w L_i^* + r K_i^*$$

The individual demand function for commodities $\mathbf{c}_i(\mathbf{p},w,r)$ arises from the solution of the single-period utility maximization problem subject to the constraint.

The aggregate demand for consumption goods is $\mathbf{c}(\mathbf{p},w,r) := \Sigma_i \mathbf{c}_i(\mathbf{p},w,r)$.

VIII.1 Equilibrium conditions: traditional vs neo-Walrasian model

	Traditional model	**neo-Walrasian model**
Market clearing conditions	$\mathbf{c}(\mathbf{p},w,r) = \mathbf{q} - \mathbf{x}$	$\mathbf{c}(\mathbf{p},w,r) = \mathbf{q} - \mathbf{x}$
	$L = L^*$	$L = L^*$
	$\mathbf{x}^{\mathrm{T}}\mathbf{p} = K^*$	

First order conditions

$$p_n - \lambda \frac{\partial \varphi(\mathbf{q},\mathbf{x},L)}{\partial q_n} = 0 \qquad n = 1, 2, \ldots, N$$

$$-p_n (1 + r) - \lambda \frac{\partial \varphi(\mathbf{q},\mathbf{x},L)}{\partial x_n} = 0 \qquad n = 1, 2, \ldots, N$$

$$-w - \lambda \frac{\partial \varphi(\mathbf{q},\mathbf{x},L)}{\partial L} = 0$$

$$\varphi(\mathbf{q},\mathbf{x},L) = 0$$

	Traditional model	neo-Walrasian model
Zero net acc. condition		$s(\mathbf{p},w,r) - \mathbf{x}^{\mathrm{T}}\mathbf{p} = 0$

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Zero net acc. condition

$$s(\mathbf{p},w,r) - \mathbf{x}^T\mathbf{p} = 0$$

IX.1 A different idea of stationary state

In the neo-Walrasian model, the individual gross savings come from intertemporal utility maximization. They are $s_i(\mathbf{p},w,r) := w - [\mathbf{c}_{i1}(\mathbf{p},w,r)]^{\mathrm{T}}\,\mathbf{p}$.

Accordingly, if $s(\mathbf{p},w,r) - \mathbf{x}^{\mathrm{T}}\mathbf{p} = 0$, then we know for sure that the equilibrium levels of $\mathbf{p}$, w and r do not involve any inducement to net capital accumulation.

By contrast, in the traditional model, the amount of individual gross savings K_i^* is just a given magnitude, i.e. it does not result from any utility maximization.

Therefore, we cannot exclude that, at the equilibrium levels of $\mathbf{p}$, w and r, a tendency to net capital accumulation will emerge when intertemporal decisions—and not just single-period decisions—are allowed for.

IX.2 A different idea of stationary state

The traditional long-run equilibrium can accordingly be considered a *quasi-stationary* state: capital accumulation is not rigidly excluded, but simply neglected.

For this reason, Garegnani and some other scholars maintained that the quasi-stationary equilibrium—unlike the real stationary state—cannot be considered as a special and practically irrelevant case. According to these authors, the quasi-stationary equilibrium must be regarded as a neoclassical re-interpretation of Adam Smith’s idea of a theoretical position toward which actual prices and distribution variables tend to gravitate.

IX.3 A different idea of stationary state

As a result, the condition $\mathbf{x}^{T}\mathbf{p} = K^{*}$ is still a stationarity condition, despite the fact that it seems symmetrical to the equilibrium condition between demand for and supply of labour.

Since that equation looks like a market-clearing condition, it seems to suggest that there is a market for capital as well as a market for labour, corroborating the idea that capital is another factor of production, on the same footing as labour, which accordingly receives a payment that reflects its productive contribution.

All that, however, is just the result of the misinterpretation toward which the founders of the neoclassical approach pointed their followers.

X.1 Conclusions

I) The neo-Walrasian model is able to determine the vector of stationary relative prices **p** and the distribution variables *r* and *w* without the need to consider capital as a factor of production and the rate of interest as the price for its use.

II) Contrarily to what some scholars have maintained, the zero net-accumulation condition is not missing in the traditional model with a given stock of capital. However, it is modified so as to appear similar to a market clearing condition.

III) Once the zero net-accumulation condition is reformulated as the equality between the given stock of 'existing capital' and the value of capital goods employed by firms, capital accumulation is not rigidly excluded, but simply ignored: savings correspond to the value of the existing capital by assumption and not as a result of households' utility optimization. However, as we know, this way of conceiving the zero net-accumulation condition leads the neoclassical theory of value to a circular reasoning.

Thank you!