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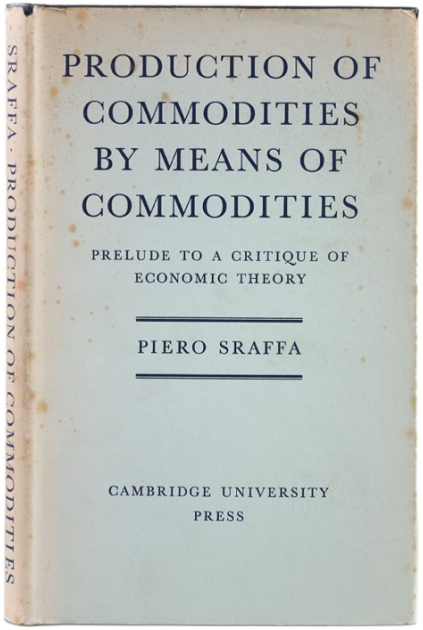

Sraffa and the theory of value

PART TWO

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6.1 PCMC: production for subsistence (chapter I)

"Let us consider an extremely simple society which produces just enough to maintain itself. Commodities are produced by separate industries and are exchanged for one another at a market held after the harvest." (Sraffa 1960, p. 3)

$$280 \text{ qr. wheat} \oplus 12 \text{ t. iron} \rightarrow 400 \text{ qr. wheat}$$

$$120 \text{ qr. wheat} \oplus 8 \text{ t. iron} \rightarrow 20 \text{ t. iron}$$

"each commodity, which initially was distributed between the industries according to their needs, is found at the end of the year to be entirely concentrated in the hands of its producer." (Sraffa 1960, p. 3)

6.2 PCMC: production for subsistence (chapter I)

"There is a unique set of exchange-values which if adopted by the market restores the original distribution of the products and makes it possible for the process to be repeated; such values spring directly from the methods of production" (Sraffa 1960, p. 3)

$$280 \text{ qr. wheat} \oplus 12 \text{ t. iron} \rightarrow 400 \text{ qr. wheat}$$

$$120 \text{ qr. wheat} \oplus \quad 8 \text{ t. iron} \rightarrow 20 \text{ t. iron}$$

120 qr. of wheat are exchanged for 12 t. of iron. Accordingly:

$$\frac{p_i}{p_w} = \frac{120}{12} = 10$$

6.3 PCMC: production for subsistence (chapter I)

An example with three commodities (Sraffa 1960, p. 4):

$$240 \text{ qr. wheat} \oplus 12 \text{ t. iron} \oplus 18 \text{ pigs} \rightarrow 450 \text{ qr. wheat}$$

$$90 \text{ qr. wheat} \oplus 6 \text{ t. iron} \oplus 12 \text{ pigs} \rightarrow 21 \text{ t. iron}$$

$$120 \text{ qr. wheat} \oplus 3 \text{ t. iron} \oplus 30 \text{ pigs} \rightarrow 60 \text{ pigs}$$

"while in the two-industry system the amount of iron used in wheat-growing was necessarily of the same value as the amount of wheat used in iron-making, this, when there are three or more products, is no longer necessarily true of any pair of them. Thus in the last example there is no such equality and the replacement can only be effected thorough triangular trade." (Sraffa 1960, p. 4)

6.4 PCMC: production for subsistence (chapter I)

Prices are determined as the solution of a system of equations:

$$240\, p_w + 12\, p_i + 18\, p_p = 450\, p_w$$

$$90\, p_w + 6\, p_i + 12\, p_p = 21\, p_i$$

$$120\, p_w + 3\, p_i + 30\, p_p = 60\, p_p$$

There are 3 equations, but just 2 are independent equations. The system can univocally determined two relative prices:

$$\frac{p_i}{p_w} = 10 \text{ and } \frac{p_g}{p_w} = 5$$

6.5 PCMC: production for subsistence (chapter I)

Let us assume (Sraffa 1960, p. 4) there are k commodities, labelled "a", "b", ..., "k".

$$
\begin{aligned}
&A_a p_a + B_a p_b + \cdots + K_a p_k = A p_a \\
&A_b p_a + B_b p_b + \cdots + K_b p_k = B p_b \\
&\vdots \\
&A_k p_a + B_k p_b + \cdots + K_k p_k = K p_k
\end{aligned}
$$

With: $A = A_a + A_b + \cdots + A_k$; $B = B_a + B_b + \cdots + B_k$; ...; $K = K_a + K_b + \cdots + K_k$. The system is in a "self-replacing state".

7.1 PCMC: production with a surplus (chapter II)

"If the economy produces more than the minimum necessary for replacement [then] there is a surplus to be distributed, …

The difficulty cannot be overcome by allotting the surplus *before* the prices are determined, as is done with the replacement of raw materials, subsistence, etc. This is because the surplus (or profit) must be distributed in proportion to the means of production (or capital) advanced in each industry; and such a proportion between two aggregates of heterogeneous goods (in other words, the rate of profits) cannot be determined before we know the prices of the goods." (Sraffa 1960, p. 6, emphasis in the original)

$$r = \frac{profit}{capital}$$

7.2 PCMC: production with a surplus (chapter II)

"On the other hand, we cannot defer the allotment of the surplus till after the prices are known, for, as we shall see, the prices cannot be determined before knowing the rate of profits. The result is that the distribution of the surplus must be determined thought the same mechanism and at the same time as are the prices of commodities." (Sraffa 1960, p. 6)

$$
\begin{aligned}
&(A_a p_a + B_a p_b + \cdots + K_a p_k)(1+r) = A p_a \\
&(A_b p_a + B_b p_b + \cdots + K_b p_k)(1+r) = B p_b \\
&\vdots \\
&(A_k p_a + B_k p_b + \cdots + K_k p_k)(1+r) = K p_k
\end{aligned}
$$

$$
\begin{gathered}
[A - (A_a + A_b + \cdots + A_k)]p_a + [B - (B_a + B_b + \cdots + B_k)]p_b + \cdots \\
+[K - (K_a + K_b + \cdots + K_k)]p_k = 1
\end{gathered}
$$

7.3 PCMC: production with a surplus (chapter II)

Basic and non-basic commodities

"One effect of the emergence of a surplus must be noticed. Previously, all commodities ranked equally, each of them being found both among the products and among means of production; as a result each, directly or indirectly, entered the production of all the others, and each played a part in the determination of prices. But now there is room for a new class of 'luxury' products which are not used, whether as instruments of production or as articles of subsistence, in the production of the others.

These products have no part in the determination of the system. Their role is purely passive. If an invention were to reduce by half the quantity of each of the means of production which are required to produce a unit of a 'luxury' commodity of this type, the commodity itself would be halved in price, but there would be no further consequence; the price-relations of the other products and the rate of profits would remain unaffected." (Sraffa 1960, pp. 7, 8)

7.4 PCMC: production with a surplus (chapter II)

The wage rate

"We have up to this point regarded wages as consisting of the necessary subsistence of the workers and thus entering the system on the same footing as the fuel for the engines or the feed for the cattle. We must now take into account the other aspect of wages since, besides the ever-present element of subsistence, they may include a share of the surplus product." (Sraffa 1960, p. 9)

A new variable w is introduced in the system. It is the wage rate, namely the wage for a unit of labour employed.

7.5 PCMC: production with a surplus (chapter II)

Assumptions:

1. "the wage is paid *post factum* as a share of the annual product, thus abandoning the classical economists' idea of a wage 'advanced' by capital." (Sraffa 1960, p. 10)

2. "We call L_a, L_b, …, L_k the annual quantities of labour respectively employed in the industries producing A, B, …, K and we define them as fractions of the total annual labour of society, which we take as unit, so that $L_a + L_b + \ldots + L_k = 1$." (Sraffa 1960, p. 10)

7.6 PCMC: production with a surplus (chapter II)

Sraffa's price equations (Sraffa 1960, p. 11)**:**

$$
\begin{aligned}
&(A_a p_a + B_a p_b + \cdots + K_a p_k)(1+r) + L_a w = A p_a \\
&(A_b p_a + B_b p_b + \cdots + K_b p_k)(1+r) + L_k w = B p_b \\
&\vdots \\
&(A_k p_a + B_k p_b + \cdots + K_k p_k)(1+r) + L_k w = K p_k
\end{aligned}
$$

$$
\begin{gathered}
[A - (A_a + A_b + \cdots + A_k)]p_a + [B - (B_a + B_b + \cdots + B_k)]p_b + \cdots \\
+[K - (K_a + K_b + \cdots + K_k)]p_k = 1
\end{gathered}
$$

Once the wage rate is fixed (independent variable), the system has k+1 equations and k+1 unknowns: the price of the k commodities in terms of the NNP and the rate of profits r.

8.1 The "value theory of labour"

NNP = total wages + total profit = 1 (because NNP is the numéraire)

w = wage rate = total wages (because $L_a + L_b + \ldots + L_k = 1$)

Proposition: If $w = 1$, then $r = 0$.

Proof: $w = 1 \Rightarrow w = \text{NNP} \Rightarrow \text{total profit} = 0 \Rightarrow r = 0$ ■

If we solve the system for w = 1, then p_a, p_b, …, p_k are equal to the quantities of labour embodied into one unit of commodity "*a*", "*b*", …, "*k*" respectively.

8.2 The "value theory of labour"

Sraffa writes (1960, p. 12, emphasis added):

> When we make w equal to 1 the whole national income goes to wages and r is eliminated. We thus revert, in effect, to the system of linear equations from which we started, with the difference that the quantities of labour are now shown explicitly intead of being represented by quantities of necessaries fro subsistence.
>
> At this level of wages the relative values of commodities are in proportion to their labour cost, that is to say to the quantity of labour which directly and indirectly has gone to produce them. **At no other wage-level do values follow a simple rule**.

8.3 The "value theory of labour"

Let us assume $p_a{}'$, $p_b{}'$, ..., $p_k{}'$ are the solution of the system for w = 1.

Now, let us imagine w = 0.9, why should the prices change? Because the proportions in which labour and means of production are employed are different in the various industries.

$$\frac{L_a}{A_a p_a{}' + B_a p_b{}' + \cdots + K_a p_k{}'} \neq \frac{L_b}{A_b p_a{}' + B_b p_b{}' + \cdots + K_b p_k{}'} \neq \cdots$$

8.4 The "value theory of labour"

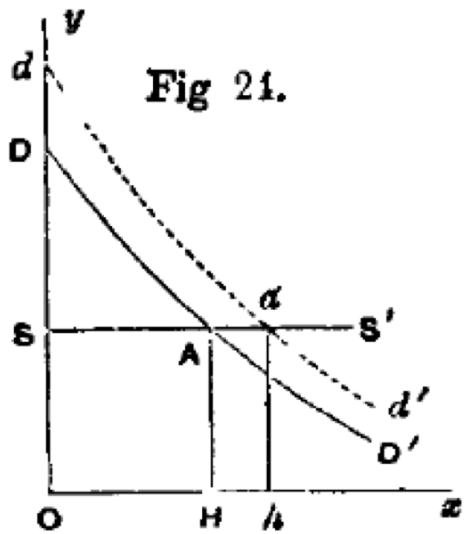
Sraffa writes (1960, pp. 12, 13):

> It is clear that if the proportions were the same in all industries non price-change could ensue, however great was the diversity of the commodity-composition of the means of production in different industries. For in each industry an equal deduction from the wage would yield just as much as was required for paying the profits on its means of production at a uniform rate without need to disturb the existing prices.
>
> For the same reason it is impossible for the price to remain unchanged when there is inequality of 'proportions'.

Therefore, relative prices do not correspond, in general, with the relative quantities of labour embodies.

9.1 The Conclusion

In his article of 1926, proving that only the assumption of constant returns to scale is consistent with Marhall's equilibrium approach, Sraffa believed that he had rediscovered the theory of the classical economists.



However, in this way he was (erroneously) attributing to the classical economist a determination of commodity prices based on the equilibrium between supply and demand.

9.2 The Conclusion

When – in the late 1927 – Sraffa re-discovered the "physical real cost" he understood that commodity prices can be determined without a supply-and-demand apparatus.

Actually, in *Production of Commodities by Means of Commodities*, commodity prices and the rate of profit are determined on the following given magnitudes:

i) Gross outputs (A, B, …, K)

ii) Quantities of labour and commodities employed (L_a, A_a, B_a, …, K_a, L_b, A_b, B_b, …, K_b, …, L_k, A_k, B_k, …, K_k)

iii) Wage rate (w)

9.3 The Conclusion

As Sraffa (1960, p. v) wrote in the *Preface* of his book:

> Anyone accustomed to think in terms of equilibrium of demand and supply may be inclined, on reading these pages, to suppose that the argument rests on a tacit assumption of constant returns in all industries. If such a supposition is founded helpful, there is no harm in the reader's adopting it as a temporary working hypothesis. In fact, however, no such assumption is made. No change in output and no change in the proportions in which different means of production are used by an industry is considered, so that no question arises as to the variation or constancy of returns.

9.4 The Conclusion

And in the following page (Sraffa 1960, p. vi):

> The temptation to presuppose constant returns is not entirely fanciful. It was experienced by the author himself when he started on these studies many years ago—and it led him in 1925 into an attempt to argue that only the case of constant returns was generally consistent with the premises of economic theory.

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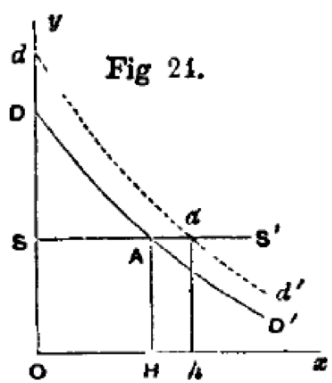

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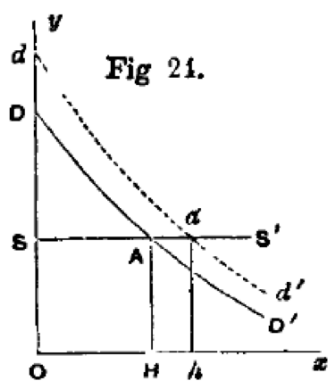

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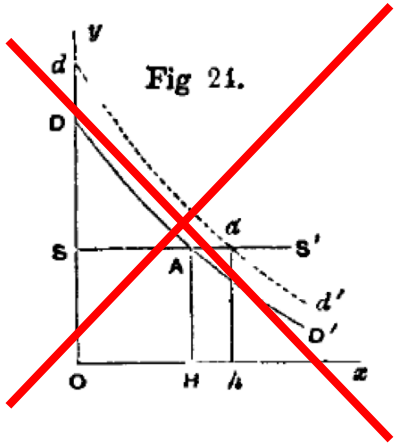

Turning Point

$$(A_a p_a + B_a p_b + \ldots + K_a p_k)(1+r) + L_a w = A p_a$$
$$(A_b p_a + B_b p_b + \ldots + K_b p_k)(1+r) + L_b w = B p_b$$
$$\cdots\cdots\cdots\cdots$$
$$(A_k p_a + B_k p_b + \ldots + K_k p_k)(1+r) + L_k w = K p_k$$

$$[A-(A_a+A_b+\ldots+A_k)]\,p_a + [B-(B_a+B_b+\ldots+B_k)]\,p_b + \ldots + [K-(K_a+K_b+\ldots+K_k)]\,p_k = 1$$